International Transmission of Credit Shocks in an Equilibrium Model with Production Heterogeneity

Yuko Imura
Bank of Canada

Julia K. Thomas*
Ohio State University

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ABSTRACT

Many policymakers and researchers view the recent financial and real economic crises in North America, Europe and beyond as a global phenomenon. Some have argued there was a common source: the U.S. financial crisis. In light of the great trade collapse of 2008-2009, and guided by the notion that large, credit-induced reductions in investment might sharply depress import demand, this paper investigates the extent to which a credit shock in one country is transmitted internationally through goods trade alone.

We develop a quantitative two-country equilibrium business cycle model wherein intermediate-good producers face persistent idiosyncratic productivity shocks and occasionally binding collateral constraints for investment loans. Our model predicts that a negative credit shock in one country induces a sharp contraction at home driven by increased misallocation. However, absent financial spillover, the resulting recession in its trading partner is both mild and lagged.

Because a financial shock prompts disproportionate declines in the affected country's investment, it delivers larger reductions in international trade and greater propagation than a real shock implying the same fall in measured productivity. With lower home bias in international trade, or more substitutable traded goods, the domestic recession following a credit shock is less severe, while international transmission is greater. Nonetheless, transmission through goods trade is limited by the average trade share, which we find insufficient to deliver a sizable recession abroad. Moreover, in contrast to the roughly contemporaneous GDP declines observed starting in 2008, an isolated credit shock in our model initially implies a mild expansion and increased borrowing abroad, driven by a short-lived rise in intermediate goods exports from the affected country. These results from our model suggest that the U.S. financial crisis could have caused the global recession and trade collapse of the late 2000s only through financial contagion delivering credit crunches virtually simultaneously across a wide group of countries.

Keywords: Credit crisis, great trade collapse, collateral constraints, capital misallocation

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1 Introduction

Beginning in late 2007, advanced economies across North America, Europe and beyond experienced severe and persistent financial and real economic crises. A high degree of business cycle synchronization across these countries in subsequent years has led policymakers, analysts and researchers to view the crises as a global phenomenon. Many have argued that the widespread, concurrent recessions over this unusual episode had a common source: the U.S. financial crisis.

There are various channels through which a financial crisis in one country could in principal induce economic slowdowns abroad. A series of empirical studies have considered both real and financial factors that may have contributed to the synchronization of economic activity since the onset of the U.S. financial crisis. One striking observation from this episode is that the global recession was accompanied by a sharp collapse of international trade in goods, with real world trade falling roughly 15 percent between 2008Q1 and 2009Q1. While globalization of financial markets may have played a crucial role in accelerating the economic downturn across countries, the synchronization of trade contractions suggests that goods trade and the resulting exposure of countries to external shocks may also have contributed to their macroeconomic responses and propagated the global recession in an important way.

A number of recent studies examining closed-economy business cycle models have found that financial shocks can cause large, persistent recessions. To the extent that a financial shock in one country drives unusually large declines in domestic investment, it may also lead to sharp reductions in that country’s import demand. Given the coincident timing of the U.S. financial crisis and the trade collapse of 2008 - 2009, it is then natural to consider the extent to which a financial shock in one country is transmitted through goods trade in an international business cycle model. We examine this question using a two-country model with intermediate goods trade that is in other respects similar to the closed-economy general equilibrium model of Khan and Thomas (2013).

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1 Imbs (2010) finds that the business cycle synchronization among OECD countries is associated with external bank lending, while the trade channel is more important for non-OECD countries. Lane and Milesi-Ferretti (2011) report that the pre-crisis levels of GDP per capita, growth in GDP and private credit, current account deficits, and trade openness are significantly correlated with the intensity of the recent crisis. Rose and Spiegel (2011) provide some evidence that current account, credit market regulation and credit growth are significant indicators of the crisis, although their significance depends on the sample of countries and measures of the crisis.

2 Lane and Milesi-Ferretti (2011) find that openness to international trade had significant effects on the severity of affected countries’ recessions, and that an individual country’s GDP movement was affected by the coincident economic performance of its trade partners.

Intermediate-good producers in our economy are heterogeneous in their capital, debt and productivity. These firms experience persistent idiosyncratic productivity shocks each period, in addition to country-specific aggregate shocks. While they may take on one-period loans from domestic households to finance their investment in physical capital, these loans are limited by collateralized borrowing constraints that depend on their individual cash holdings. Countries are connected with each other through two channels. First, intermediate goods are traded across countries, and imports are combined with domestic intermediate goods to produce final goods used for consumption and investment. Second, households trade state-contingent one-period bonds in complete international financial markets. We calibrate the parameters of our model governing firms’ decisions on investment and borrowing to match key aspects of establishment-level investment data and the average aggregate debt-to-asset ratio in the United States.

Our model predicts that a financial shock in one country will induce an immediate, sharp contraction in the domestic economy, followed by a mild, but persistent downturn in its trade partner. When the availability of credit is suddenly tightened in one country, borrowing across domestic intermediate goods producers is curtailed, causing a sharp fall in aggregate investment and reducing domestic production. Interestingly, while exports ultimately experience a persistent fall, the series briefly rises at the onset of the shock. At the heart of the story is the general equilibrium consequence of an endogenous decline in aggregate productivity stemming from cash-poor firms’ reduced ability to access the loans they need to finance efficient investment. The impending increased misallocation of production makes domestic households feel both poorer and less inclined to save, reducing the demand for final goods. That, in turn reduces demand for intermediate goods, depressing domestic GDP, wages and employment, as well as the demand for imports. Given their predetermined capital stocks, domestic firms’ production of intermediates is initially high relative to domestic demand. The excess is exported at temporarily low prices.

With the affected country’s release of intermediates for cheap export, its trading partner experiences a brief rise in economic activity. However, as misallocation worsens in the country with tight credit, its exports start falling, while its import demand continues downward. At that point, the trade partner’s recession begins. Reduced demand for its exports discourages investment and employment; this compounds the negative effects of increased import prices, reducing final goods production and thus the production of intermediates. Quantitatively, however, the recession abroad is far smaller than it is in the country directly experiencing the shock so long as our model is calibrated to reproduce the average trade share in the U.S. If the trade share is counterfactually large, international transmission is more pronounced; however, so is the initial upturn.
The degree to which a credit-driven recession is propagated abroad through goods trade depends on the extent of home bias in international trade and the type of goods countries trade with each other. Lower home bias dampens the domestic recession, but amplifies international transmission. When the weight on imported goods in final production is larger, each country is more susceptible to the health of its trade partner and less to shocks in its own economy. For the country experiencing the credit shock, the impact on domestic production is mitigated; for the trade partner, a larger reliance on imports in final production implies greater effects of the shock abroad for its own economy. In sum, the more important is international goods trade, the larger are the effects of a credit shock in one country on the economies of its trade partners. This result is consistent with Lane and Milesi-Ferretti’s (2011) empirical finding of a significant positive correlation between countries’ pre-crisis levels of openness to international trade and the depth of their respective recessions. Similarly, when traded goods are less substitutable across countries, the domestic recession is less severe following a credit shock, while international transmission is greater. When domestically produced intermediate goods are not easily replaced by imports, the high reliance on domestic goods mitigates the fall in domestic intermediates production, dampening the effects of the credit shock for domestic investment and employment. On the other hand, for the trading partner, final good production falls by more, as do investment and consumption, since the decline in imports from the directly affected country is not easily replaced with local products. This result is consistent with Heathcote and Perri’s (2002) finding in a two-country business cycle model that the international comovement of output is decreasing in the cross-country elasticity of substitution under complete international financial markets.

As in Khan and Thomas (2013), tightened collateral constraints have disproportionately negative effects on the investment decisions of firms with relatively high productivity and little cash. This implies an inefficiently low allocation of capital to these firms, distorting the allocation of production in coming periods, and thus generating an endogenous fall in measured total factor productivity. Because the increased misallocation does not happen at once, but instead worsens fairly predictably over time, the consequences for aggregate investment are disproportionately large. This amplifies international transmission. Comparing those responses from our model economy to responses when one country experiences an exogenous productivity shock implying a fall in measured TFP of equal size and persistence, we find that a credit shock induces substantially larger ultimate declines in trade volumes and a far more severe recession abroad; peak-to-trough declines in exports and foreign GDP are roughly twice as large when compared to those induced by the exogenous productivity shock. This prediction from our model is consistent with empirical evidence.
linking financial crisis to the great trade collapse of 2008 - 2009. These points notwithstanding, the recession transmitted abroad is too mild and too delayed to explain the recent data reviewed below, leading us to reject a pure goods trade explanation. On balance, our model indicates that a financial crisis can cause a global recession and trade collapse only through financial contagion delivering credit crunches virtually simultaneously across a wide group of countries.

The remainder of the paper is organized as follows. Section 2 briefly reviews recent economic performance in the U.S. and other G7 countries. Section 3 discusses the literature most related to our analysis. We describe the model in section 4, and its calibration in section 5. Section 6 reports results, and section 7 concludes.

2 The U.S. financial crisis and the global recession

We begin with a review of the business cycle experiences in the U.S. and other G7 countries during and following the U.S. 2007-2009 recession, as well as recent credit conditions in the United States. Perri and Quadrini (2014) provide a more in-depth examination of these data and also analyze other postwar U.S. recession episodes for comparison; the brief summary here is merely to set the stage. Figure 1 shows log-detrended quarterly real GDP, investment, consumption and employment in the U.S. from 2007Q4 to 2013Q1, expressed as percentage deviations from their respective levels in 2007Q4 when the recession started.

By the second quarter of 2009, real GDP and consumption had fallen 5.3 percent and 4.1 percent, respectively. Investment fell sharply, reaching 14.3 percent below its 2007Q4 level by 2009Q2. Consumption and investment hovered near their trough levels for several quarters before beginning a gradual recovery in 2010. Although its decline was comparatively slow over the first several quarters of the recession, employment eventually reached 4.6 percent below its 2007Q4 level. The post-2009Q2 recovery from this sharp recession has been sluggish and uneven. As of 2013Q1, no series in Figure 1 had regained its pre-recession level.

See, for example, Behrens, Corcos and Mion (2013) and Coulibaly, Sapienza and Zlate (2011). Bems, Johnson and Yi (2012) survey studies of the collapse in international trade during the recent global recession. Taken as a whole, a series of studies suggest that the dominant force behind the trade collapse was the collapse in aggregate expenditure (Bems, Johnson and Yi (2010, 2011), Eaton et al. (2011), Bussière et al. (2013)). Alessandria et al. (2010, 2011, 2013) emphasize inventory adjustments as an important amplification mechanism.
FIGURE 1. U.S. economy and the 2007-2009 recession

NOTE.– Data from OECD Main Economic Indicators. All series are in logs, detrended using the Hodrick-Prescott filter with weight 1600, and plotted as percent deviations from 2007Q4 values. Shaded gray bar denotes recession dates defined by the N.B.E.R. Business Cycle Dating Committee.

A similar pattern of steep economic downturn and sluggish recovery is evident for other advanced economies during this period. In Figure 2, we plot log-detrended real GDP, investment, consumption and employment for G7 countries from 2007Q4 to 2013Q1. As in Figure 1, these series are percentage deviations from their respective 2007Q4 levels. The comovement in GDP and investment across these countries is striking, particularly during the U.S. recession dates. Although less synchronized across countries than GDP and investment, consumption also fell in all G7 countries until mid-2009 and had gradual recoveries until the middle of 2010. Relative to other G7 countries, the fall in the U.S. employment was distinctively large. Perri and Quadrini (2014) suggest this may be due to differences in the structures of countries’ labor markets. Nonetheless, all G7 countries experienced employment declines and sluggish employment recovery over the following years.

What could cause such severe global recession? Some have argued that it was triggered by a financial crisis in the United States. Following the collapse in housing markets starting in the mid 2000’s, it became increasingly evident by 2007 that credit market conditions had begun to deteriorate in the U.S. According to the Senior Loan Officer Opinion Survey of the Federal Reserve Board, many banks started to enforce stricter conditions on their loans in 2007, and the number of domestic banks that tightened their loan standard soared between 2007 and 2008, reaching 80 percent (in net) by the end of 2008, as seen in the left panel of Figure 3. The tighter loan standards
are reflected in a sharp decline in the growth rate of private sector debt, shown in the right panel of Figure 3. With the peak of the housing market crisis in 2006-2007, the growth rate of private sector debt plunged from 8.4 percent to -1.7 percent between 2007 and 2009.

**FIGURE 2. G7 countries and the 2007-2009 U.S. recession**

![Real GDP and Investment](image1)

![Employment and Consumption](image2)

NOTE.— Data from OECD Main Economic Indicators. All series are in logs, detrended using the HP-filter with weight 1600, and plotted as percent deviations from 2007Q4 values.

**FIGURE 3. U.S. lending standards and volume**

![Net % of domestic banks tightening loan standards and Growth rate of private sector debt](image3)

NOTE.— Shaded area reflects 2007 U.S. recession dates. Data sources: Senior Loan Officer Opinion Survey on Bank Lending Practices, Federal Reserve Board, OECD Main Economic Indicators.
3 Related literature

Our paper contributes to a large literature on the role of financial frictions in propagating business cycle fluctuations. Our particular focus on collateralized borrowing constraints as a source of frictions follows on a line that stems from the seminal work of Kiyotaki and Moore (1997). Proposing a model where durable assets serve as collateral for loans, they examine how credit constraints interact with aggregate economic activity over the business cycle, and show that the interdependence of credit limits and the prices of collateralized assets plays an important role in amplifying and propagating shocks affecting firms’ net worth.

The model we develop is a two-country extension of the financial frictions model of Khan and Thomas (2013), which introduces an endogenous TFP channel for credit shock propagation. There, as here, firms experience persistent shocks to their individual productivity levels, and they face collateral constraints when borrowing to finance their capital investment. When collateral constraints are tightened by a credit shock, the financing barriers that prevent cash-poor firms with relatively high productivities from investing to their optimal capital levels are increased. As a result, a credit shock disrupts the allocation of capital across firms, inducing an endogenous decline in aggregate productivity that, in turn, delivers a persistent decline in real economic activity.

We join a large literature on international business cycles starting with Backus, Kehoe and Kydland (1992) and Baxter and Crucini (1993, 1995). Without cross-country spillovers embedded in the shock processes, standard international business cycle models with trade in goods and bonds routinely fail to translate a recession in one country into a significant recession in its trading partner. Given the strong propagation mechanism that collateral constraints and firm heterogeneity have been seen to deliver in closed-economy settings, we explore whether the combination of these elements in a two-country business cycle model might overcome this difficulty. In light of the great trade collapse during the recent financial crisis, we examine whether these new propagating forces can produce strong international transmission of financial shocks.

Our focus on trade linkages as a potential source of international comovement is related to the analysis by Kose and Yi (2006), who assess whether a standard international business cycle framework can account for the high correlation of business cycles for countries with strong trade ties. While their model does imply that international correlations grow with the extent of inter-

5 See, for example, Bernanke and Gertler (1989), Aiyagari and Gertler (1999), Bernanke, Gertler and Gilchrist (1999), Kocherlakota (2000), and Cooley, Marimon and Quadrini (2004).
national trade, its predicted change in the cross-country GDP correlation for a given change in trade intensity is significantly smaller than that in the data. We do not expressly measure our model-generated elasticity of international comovement with respect to trade linkages; however, we show that stronger trade relationships increase international transmission of financial shocks.

Our paper is also related to recent studies examining the relationship between financial integration and international business cycle comovement in quantitative frameworks emphasizing the role of financial frictions in propagating aggregate shocks across countries. Devereux and Yetman (2010) develop a two-country model with international portfolio holdings wherein investors borrow from savers in order to invest in domestic and foreign fixed assets (equity), but their borrowing is limited by the value of their equity. Portfolio diversification by investors implies that asset prices are positively correlated across countries, and hence a negative productivity shock lowering the asset price in one country generates a tightening of borrowing constraints in both countries. This hinders investment in fixed assets used in final-good production in both countries, delivering international comovement in production.

Using a similar framework, Devereux and Sutherland (2011) show that an exogenous tightening of the leverage constraint also generates positive cross-country comovement of macroeconomic variables when equity markets are internationally integrated. More recently, Devereux and Yu (2014) extend the framework to allow for occasionally-binding collateral constraints; they show that moving from financial autarky to financial integration not only increases the probability that collateral constraints bind in one country, but also leads these constraints to bind simultaneously in both countries, thereby increasing cross-country comovement.

Dedola and Lombardo (2009) pursue an alternative approach to our emphasis on collateralized borrowing limits. They develop an endogenous portfolio-choice model exploring the financial accelerator channel of Bernanke, Gertler and Gilchrist (1999), wherein investors’ borrowing costs depend on an external finance premium that falls in their net worth. They show that the cross-country equalization of credit spreads due to international financial integration leads to strong comovements in asset prices and real activity regardless of the degree of exposure to foreign assets.

The recent financial integration studies above highlight the presence of international investors with access to foreign assets as an important channel through which country-specific shocks are transmitted across countries. With international financial integration, financial conditions in two countries become directly interdependent, so that country-specific shocks induce strong cross-country comovement. As noted above, we focus instead on international goods trade, exploring the effects of reduced production capacity among financially constrained firms, and how the resulting
misallocation and supply shortages affect the economies of a country’s trade partners.

Perri and Quadrini (2014) introduce a global self-fulfilling liquidity shortage as an explanation for international comovement during the recent global recession. In addition to a Kiyotaki and Moore style borrowing constraint applying to the finance of working capital requirements, they assume that firms can purchase capital of liquidated firms at a high price only if their borrowing constraints are not binding. Otherwise, the liquidated capital is sold to households at a low price. In this environment, the price of liquidated capital becomes self-fulfilling, and the economy has multiple equilibria, with the price of capital switching stochastically between low and high states. International financial integration equalizes the prices of liquidated capital across countries and leads the borrowing constraints to bind simultaneously in the two countries, thus generating international comovements in real and financial variables.

In contrast to the setting in Perri and Quadrini (2014), firms in our model are owned by domestic households, so firms’ stochastic discount factors are not necessarily equalized across countries. As mentioned above, our firms are heterogeneous in their capital, debt, and productivity. The tightness of their borrowing constraints in any given date depends both on aggregate credit conditions within their country and their individual levels of cash-on-hand, where the latter is jointly determined by the worldwide aggregate state vector and the three individual state variables that distinguish them.

4 Model

We assume two symmetric countries, country 1 and country 2. In each country, there is a continuum of identical infinitely-lived households, each with access to state-contingent nominal bonds, and a representative final-good producer that combines domestically-produced intermediate goods and imported intermediate goods to produce a final good used for domestic consumption and capital investment. Each country’s intermediate good is produced by a unit measure of heterogenous domestic firms. All markets are perfectly competitive, and all prices are flexible.

Intermediate good firms sell their output domestically and abroad. They produce with capital and labor, and they face persistent country-specific aggregate total factor productivity shocks and persistent firm-specific productivity shocks. Firms hire labor from domestic households, but maintain their own capital stocks. Each firm buys investment goods from the final-good producer in its country to augment its capital for the next period, and each can access one-period loans to help finance these purchases. A collateralized borrowing constraint in each country limits the debt any firm can take on as a function of its cash. Firms cannot circumvent the constraint by paying
negative dividends. We also assume exit and entry at an exogenous rate each period to prevent all firms effectively outgrowing financial frictions in the long run.

We represent the aggregate state of the world economy by \( A \), where \( A \equiv (Z, S) \). The exogenous state vector is \( Z \), where \( Z \equiv [z_1, z_2, \theta_1, \theta_2] \). Its first two elements represent aggregate productivity in country \( c \), for \( c = 1, 2 \). The last two elements represent credit states; each \( \theta_c \) parameterizes a country-specific collateral constraint limiting firms’ debt in proportion to their cash. All exogenous state variables are assumed to follow Markov chains.

Our model generates a time-varying distribution of firms over capital, \( (k \in K \subset \mathbb{R}_+) \), debt \( (b \in B \subset \mathbb{R}) \) and firm-specific productivity \( (\varepsilon \in E) \) in each country. We summarize the distribution of firms at the start of a period in country \( c \) using the probability measure \( \mu_c \) defined on the Borel algebra \( S \) generated by the open subsets of the product space, \( S = K \times B \times E \) for each \( c = 1, 2 \). The endogenous aggregate state vector in our model is \( S \equiv [\mu_1, \mu_2, B_1, B_2] \), where \( B_1 \) and \( B_2 \) represent the state-contingent bonds held by households in each country at the start of the period. All agents in the economy take as given the laws of motion determining \( Z' \) given \( Z \), as well as the evolution of the endogenous state according to an equilibrium mapping \( S' = \Gamma(A) \). We describe the preferences, technologies and optimization problems for country 1 below, specifying the country 2 counterparts only where necessary for clarity or in defining notation.

4.1 Households

The representative household in each country is endowed one unit of time in each period, and values its consumption and leisure according to a period utility function \( u(C, 1 - N) \). Future utility is discounted by the subjective discount factor \( \beta \in (0, 1) \). Household wealth is held in three forms. First, there are one-period shares in domestic firms, which we identify using the measures \( \zeta_c \) for \( c = 1, 2 \). Next, there are one-period noncontingent real bonds corresponding to the total debts of all domestic firms, which we denote by \( \phi_c \) for \( c = 1, 2 \). Finally, as noted above, households have access to a complete set of state-contingent nominal bonds. Those bonds are denominated in units of the country 1 currency, and we use \( B_c(A) \) to denote the nominal bonds with which the household in country \( c \) enters the period given current aggregate state \( A \).

The household in country 1 chooses its consumption, \( C_1 \), the hours of labor it supplies to firms, \( N_1 \), its shares in firms of each type with which to begin the next period, \( \zeta'_1(k', b', \varepsilon') \), and its real bonds for next period, \( \phi'_1 \). The household also chooses its state-contingent nominal bonds, \( B_1(A') \), which each promise delivery of one unit of country 1 currency if the state \( A' \) is realized next period. Let \( g(A'; A) \) be the real price of one such bond, denominated in units of country 1
consumption goods. Next, let the dividend-inclusive values of the household’s current firm shares be \( \tilde{p}_1(k, b, \varepsilon; A) \), and the ex-dividend prices of new shares in a given firm type be \( p_1(k', b', \varepsilon'; A) \). Let \( q_1(A) \) be the country 1 consumption goods the household must forfeit per unit real bond, let \( w_1(A) \) be the domestic real wage, and let \( P_1(A) \) be the domestic aggregate price level. Finally, let \( G(A'|A) \) represent the conditional probability of realizing given state \( A' \) next period, which will be determined by \( S' = \Gamma(A) \) and the exogenous transition probabilities for the elements of \( Z \). Given this notation, the country 1 household’s expected lifetime utility maximization problem can be written as follows.

\[
V^h_1(\zeta_1, \phi_1, B_1(A); A) = \max_{C_1, N_1, \zeta'_1, \phi'_1, B_1(A')} u(C_1, 1 - N_1) + \beta \int V^h_1(\zeta'_1, \phi'_1, B_1(A'); A') G(dA'|A) \quad (1)
\]

subject to:

\[
\int \tilde{p}_1(k, b, \varepsilon; A) \zeta_1 (d [k \times b \times \varepsilon]) + \phi_1 + \frac{B_1(A)}{P_1(A)} + w_1(A)N_1 \geq C_1 + \int p_1(k', b', \varepsilon'; A) \zeta'_1 (d [k' \times b' \times \varepsilon']) + q_1(A)\phi'_1 + \int g(A'; A)B_1(A')dA' \quad (2)
\]

Let \( \lambda_1(A) = D_1u(C_1, 1-N_1) \) be the Lagrange multiplier on the budget constraint in the problem above. The household’s efficiency conditions with respect to hours worked, firm shares, and real bonds immediately imply a series of restrictions on the country 1 real wage, firm share prices and inverse loan price listed in (2) - (4). Its efficiency conditions with respect to state-contingent nominal bonds yield the additional price restrictions in equation 5.

\[
w_1(A) = \frac{D_2u(C_1, 1-N_1)}{\lambda_1(A)} \quad (2)
\]

\[
p_1(k', b', \varepsilon'; A) = \int \frac{\beta \lambda_1(A')}{\lambda_1(A)} \tilde{p}_1(k', b', \varepsilon'; A') G(dA'|A) \quad (3)
\]

\[
q_1(A) = \int \frac{\beta \lambda_1(A')}{\lambda_1(A)} G(dA'|A) \quad (4)
\]

\[
g(A'; A) = \frac{\beta \lambda_1(A')}{\lambda_1(A)} \frac{1}{P_1(A')} G(A'|A) \quad (5)
\]

The household in country 2 solves an analogous problem adjusted for the fact that the nominal bonds it holds are denominated in the other country’s currency. Let \( Q(A) \) represent the current real exchange rate, the price of country 2 final output in units of country 1 final output. Each nominal bond held at the start of the period returns \( \frac{1}{P_1(A)} \) units of country 1 output, each worth \( Q(A)^{-1} \) units of country 2 consumption goods. Similarly, one nominal bond for a given next period
state $A'$ costs the country 2 household $\varphi(A'; A)$ units of country 1 output, each implying the forfeit of $Q(A)^{-1}$ units of country 2 consumption.

$$V^h_2(\zeta_2, \phi_2, B_2(A); A) = \max_{C_2,N_2,\zeta_2',\phi_2',B_2(A')} u(C_2, 1 - N_2) + \beta \int V^h_2(\zeta_2', \phi_2', B_2(A'); A') G(dA'|A) \quad (6)$$

subject to:

$$\int \rho_2(k, b, \varepsilon; A) \zeta_2(d[k \times b \times \varepsilon]) + \phi_2 + \frac{B_2(A)}{P_1(A)Q(A)} + w_2(A)N_2 \geq C_2 + \int \rho_2(k', b', \varepsilon'; A) \zeta_2'(d[k' \times b' \times \varepsilon']) + q_2(A)\phi_2' + \int \frac{\varphi(A'; A)}{Q(A)} B_2(A')dA'$$

Let $\lambda_2(A) = D_1u(C_2, 1 - N_2)$. The country 2 household’s efficiency conditions imply restrictions on $w_2(A)$, $\rho_2(k', b', \varepsilon'; A)$ and $q_2(A)$ mirroring those in equations 2 - 4, and restrict nominal bond prices to satisfy the equations in (7).

$$\varphi(A'|A) = \frac{\beta \lambda_2(A')}{\lambda_2(A)} \frac{Q(A)}{P_1(A')Q(A')} G(A'|A) \quad (7)$$

Comparing (5) and (7), we arrive at a set of equations determining the evolution of the real exchange rate across every date and state: $Q(A') = \frac{\lambda_2(A')}{\lambda_1(A)} \frac{Q(A)}{\lambda_2(A)}$. Assuming an initial date zero in which $\frac{\lambda_1(A^0)Q(A^0)}{\lambda_2(A^0)} = 1$, we may write the real exchange rate in every period as the ratio of marginal utilities of consumption in countries 2 and 1.

$$Q(A) = \frac{\lambda_2(A)}{\lambda_1(A)} \quad (8)$$

### 4.2 Final goods production

The representative final-good producer in country 1 combines domestically produced intermediate goods, $y^{D1}$, and intermediate good exports from country 2, $y^{X2}$, to produce final goods, $H_1$, through the CES production function:

$$H_1 = \left[ \omega \left( y^{D1} \right)^{\frac{\rho - 1}{\rho}} + (1 - \omega) \left( y^{X2} \right)^{\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}}, \quad (9)$$

where $\rho$ is the elasticity of substitution between domestic goods and imports (Armington elasticity), and $\omega$ is the relative weight on home-produced goods (home bias). It sells its output at price $P_1(A)$ to households (for consumption) and to domestic intermediate-good firms (for investment).

The nominal prices associated with intermediate goods from each country are dominated in the currency of the country in which the good is sold. Let $p^{D1}(A)$ be the price of country 1 intermediate good sold in country 1, and let $p^{X2}(A)$ denote the price of the country 2 intermediate good sold in country 1, with both denominated in the country 1 currency. Taking as given these
input prices, the price of its output, $P_1(A)$, and the technology in (9), the final-good producer in
country 1 solves the static profit maximization problem in equation 10. Its resulting conditional
factor demands are listed in (11) - (12).

$$\max_{y^{D1},y^{X2}} P_1(A)H_1 - p^{D1}(A)y^{D1} - p^{X2}(A)y^{X2}$$

(10)

$$y^{D1} = \omega^\rho \left( \frac{p^{D1}(A)}{P_1(A)} \right)^{-\rho} H_1$$

(11)

$$y^{X2} = (1 - \omega)^\rho \left( \frac{p^{X2}(A)}{P_1(A)} \right)^{-\rho} H_1$$

(12)

The final-good producer in country 2 solves an analogous problem determining its conditional
factor demand for country 2 intermediate goods, $y^{D2} = \omega^\rho \left( \frac{p^{D2}(A)}{P_2(A)} \right)^{-\rho} H_2$, and imports from coun-
try 1, $y^{X1} = (1 - \omega)^\rho \left( \frac{p^{X1}(A)}{P_2(A)} \right)^{-\rho} H_2$. Given the conditional factor demands above, we retrieve the
aggregate price level (price index) in each country.

$$P_1(A) = \left[ \omega^\rho \left( \frac{p^{D1}(A)}{P_1(A)} \right)^{1-\rho} + (1 - \omega)^\rho \left( \frac{p^{X2}(A)}{P_1(A)} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

(13)

$$P_2(A) = \left[ \omega^\rho \left( \frac{p^{D2}(A)}{P_2(A)} \right)^{1-\rho} + (1 - \omega)^\rho \left( \frac{p^{X1}(A)}{P_2(A)} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

(14)

Country 1’s exports in units of country 1 final output are \( \frac{p^{X1}y^{X1}}{P_2} Q \), and its imports are \( \frac{p^{X2}y^{X2}}{P_1} \).

4.3 Intermediate goods firms

Throughout this section, we restrict attention to intermediate-good firms in country 1. As
there are no trade frictions, each firm is indifferent between selling a unit of its output domestically
versus exporting it in equilibrium. From the perspective of a country 1 firm, this means \( \frac{p^{X1}}{P_2} Q = \frac{p^{D1}}{P_1} \), so its problem can be described entirely in terms of domestic prices. Thus, the description of
the problems facing intermediate-good firms in country 2 mirrors the description here.

Each firm enters a period identified by \((k, b, \varepsilon)\), where \( k \) and \( b \) are the capital and debt levels it
selected at the end of last period, and \( \varepsilon \) is its current idiosyncratic productivity. Positive values of
\( b \) represent debt; negative values are financial savings. The firm produces using capital and labor
in a decreasing returns to scale Cobb-Douglas production function:

$$y_1 = z_1 \varepsilon k^{\alpha} n^\nu,$$

(15)

where \( z_1 \) is the aggregate productivity shock in its country, \( \alpha \in (0, 1), \nu \in (0, 1), \) and \( \alpha + \nu < 1 \).

We assume firm-specific productivity \( \varepsilon \) follows a Markov chain with \( N_\varepsilon \) realizations and transition
probabilities \( \varphi_{ij} = pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i) \), and that the aggregate productivity shock \( z_1 \) also follows a
Markov chain.
Given its capital and productivity, the domestic real wage, \( w_1(A) \), and the relative price of its output, \( \frac{p_{D1}(A)}{P_1(A)} \), the firm chooses its labor demand to solve the following static problem, subject to the production technology (15).

\[
\max_n \left( \frac{p_{D1}(A)}{P_1(A)} \right) y_1 - w_1(A)n
\]  

(16)

The firm’s labor and output decision rules follow immediately, as does its static profit defined as real sales less wage payments. Notice each of these is independent of the firm’s debt position.

\[
n_1(k, \varepsilon; A) = \left[ \nu \left( \frac{p_{D1}(A)}{P_1(A)} \right) \varepsilon z_1 k^{\alpha} \right]^{\frac{1}{1-\nu}}
\]

\[
y_1(k, \varepsilon; A) = z_1 \varepsilon k^{\alpha} n_1(k, \varepsilon; A)\nu
\]

\[
\pi_1(k, \varepsilon; A) = (1 - \nu) \left( \frac{p_{D1}(A)}{P_1(A)} \right) y_1(k, \varepsilon; A)
\]

4.3.1 cash and debt

Let \( x \) represent the \((k, b, \varepsilon)\) firm’s real cash-on-hand in units of the domestic final good; we define this variable as its static profit and non-depreciated capital net of outstanding debt.

\[
x \equiv \pi_1(k, \varepsilon; A) + (1 - \delta)k - b
\]  

(17)

The firm receives \( q_1(A) \) units of domestic final output in the current period for each unit of debt it incurs. Thus, taking on debt with face value \( b' \) delivers it a loan of size \( q_1(A)b' \). Capital accumulation is one period time to build; \( k' = (1 - \delta)k + i \), where \( i \) is investment. This implies the following budget constraint governing the firm’s choice of \( k', b' \) and current dividends, \( D \).

\[
x + q_1(A)b' \geq D + k'.
\]  

(18)

We assume the firm cannot issue new equity to finance its expenditures, \( D \geq 0 \), and that the debt it takes on is limited in proportion to its current cash by the collateral constraint:

\[
b' \leq \theta_1 x,
\]  

(19)

where \( \theta_1 \geq 0 \) is an exogenous state variable reflecting the availability of credit in country 1.

Note that we have assumed no real frictions impeding a firm’s capital adjustment. Furthermore, the collateral constraint in (19) implies that its ability to borrow is in no direct way affected by its capital or debt, but only indirectly through \( x \). As a result, the only relevant endogenous individual state variable from the perspective of the firm is its cash-on-hand, \( x \). We use this observation below to simplify the description of the firm’s intertemporal problem.
4.3.2 intertemporal problem

After production in any period, each firm realizes the outcome of a state-invariant, exogenous exit shock. At that point, fraction $\gamma \in (0, 1)$ of firms exit the economy with $k' = b' = 0$. Each exiting firm undertakes negative investment $(1 - \delta)k$ and returns its cash as dividends to domestic households as it departs. Exiting firms are replaced at the start of the next period by an equal number of new firms. Each new firm begins with zero debt, a capital stock $k_0$, and a productivity level drawn from the ergodic distribution of $\varepsilon$; thus the total investment in newly arrived firms in any period is $\gamma k_0$. We focus the remainder of this section on the intertemporal problem solved by a continuing incumbent firm.

It is convenient to impose state-contingent discount factors consistent with equilibrium in the market for firm shares (section 4.1) directly in stating each firm’s intertemporal optimization problem. Here, we assign $\Lambda_1(A)$ as the valuation a firm in country 1 assigns to its dividends, and assume the firm discounts its future value by the household subjective discount factor $\beta$. In equilibrium, $\Lambda_1(A)$ will be the domestic household’s marginal utility of consumption, $D_1 u(C_1(A), 1 - N_1(A))$. Thus, our statement of the firm’s problem below simply translates its value function from units of output to units of marginal utility.

Let $\tilde{v}_1$ represent the value of a country 1 firm just prior to the realization of the exit shock:

$$\tilde{v}_1(x, \varepsilon; A) = \gamma \Lambda_1(A)x + (1 - \gamma)v_1(x, \varepsilon; A),$$

(20)

where $v_1$ is the expected discounted value conditional on it continuing to the next period. The dividends paid by a continuing firm are immediate from (18) as a function of its $k', b'$ choice. Thus, we may write the problem of a continuing firm of type $(x, \varepsilon_i)$ as:

$$v_1(x, \varepsilon_i; A) = \max_{k', b'} \left[ \Lambda_1(A)[x + q_1(A)b' - k'] + \beta \int \phi_{ij} \tilde{v}_1(x_j', \varepsilon_j; A') G(dA'|A) \right],$$

(21)

subject to the collateral constraint in (19) and an equation determining next period’s cash as a function of the firm’s chosen capital and debt and the realization of $\varepsilon'$:

$$x_j' = x_1(k', \varepsilon_j; A') + (1 - \delta)k - b'.$$

(22)

The problem above can be simplified further by the following observations. In equilibrium, no continuing firm can increase its value by paying strictly positive dividends in the current period, since it borrows and lends at the same price its owners face, and $\Lambda_1(A) = \lambda_1(A)$. If a firm has amassed a sufficient coffer of cash to imply that its investment can never again be hindered by
inadequate external finance, then its shadow value of retained earnings matches the household valuation of dividends. That firm would be indifferent between paying zero or positive dividends, having reached Modigliani-Miller status. On the other hand, for any firm with insufficient cash to preclude the possibility that the collateral constraint (19) may bind in some future date and state, the per-unit valuation of retained earnings exceeds the domestic household’s valuation of dividends; any such firm’s value is maximized only when \( D = 0 \). Combining these observations, we see that \( D = 0 \) is an optimal dividend policy for any continuing firm. Imposing this policy in the binding budget constraint (18), we see that each firm’s choice of capital directly implies its debt, \( b' = (k' - x)/q_1(A) \). Thus, (21) - (22) above can be collapsed to a simple univariate problem:

\[
v_1(x, \varepsilon_i; A) = \max_{k'} \beta \int_1^N \varphi_{ij}\left[ \gamma A_1(A')x'_j + (1 - \gamma)v_1(x'_j, \varepsilon_j; A') \right]G(dA'|A) \tag{23}
\]

subject to \( x'_j = \pi_1(k', \varepsilon_j; A') + (1 - \delta)k' - (k' - x)/q_1(A) \)

and subject to \( k' \leq x[1 + \theta_1q_1(A)] \).

Let \( g_1(x, \varepsilon_i; A) \) represent the resulting capital decision rule for a firm in country 1, and let \( b'_1(x, \varepsilon_i; A) \) be the associated debt rule.

### 4.4 Recursive equilibrium

A recursive competitive equilibrium is a set of functions: \( \varrho, Q, \{w_c, q_c, \rho_c, \bar{\rho}_c, p^{D_c}, p^{X_c}, P_c, \Lambda_c\}_{c=1,2}, \{V^h_c, C_c, N_c, \zeta'_c, \phi'_c, B'_c, H_c, y^{D_c}, y^{X_c}\}_{c=1,2}, \{v_c, n_c, y_c, g_c, b'_c\}_{c=1,2} \) that solve household and firm problems and clear the markets for assets, labor, intermediate goods and final output in each country, as described by the following conditions.

(i) \( V^h_1 \) solves (1), \( V^h_2 \) solves (6), and \( (C_c, N_c, \zeta'_c, \phi'_c, B'_c) \) are the associated policy functions for households in each country \( c = 1, 2 \)

(ii) country 1 final good producer solves (10) given (9) with policy functions \( (H_1, y^{D_1}, y^{X_1}) \);

country 2 final good producer solves analogue problem with policy functions \( (H_2, y^{D_2}, y^{X_1}) \)

(iii) country \( c = 1, 2 \) firms solve (16) given (15), and \( (n_c, y_c) \) are the associated policy functions

(iv) \( v_c \) solves (23) with associated policy functions \( (g_c, b'_c) \), for \( c = 1, 2 \)

(v) \( \zeta'_c(k', b', \varepsilon_j, \zeta_c, \phi_c, B_c; A) = \mu'_c(k', b', \varepsilon_j; A) \), for each \( (k', b', \varepsilon_j) \in \mathcal{S} \) in country \( c = 1, 2 \)
(vi) \( \phi_c'(\zeta_c, \phi_c, B_c; A) = \int_{S} \left[ b_c'(k, b, \varepsilon; A) \right] \mu_c(d[k \times b \times \varepsilon]), \) for \( c = 1, 2 \)

(vii) \( B_1'(A', \zeta_1, \phi_1, B_1; A) + B_2'(A', \zeta_2, \phi_2, B_2; A) = 0 \) for all \( (A'; A) \)

(viii) \( N_c(\zeta_c, \phi_c, B_c; A) = N_c^F(A), \) where \( N_c^F(A) = \int_{S} n_c(k, \varepsilon; A) \mu_c(d[k \times b \times \varepsilon]), \) for \( c = 1, 2 \)

(ix) \( C_c(\zeta_c, \phi_c, B_c; A) = H_c(A) - I_c(A), \) where \( H_c(A) = \left( \omega [y^{DC}(A)]^{\frac{\psi - 1}{\psi}} + (1 - \omega) [y^{Xc}(A)]^{\frac{\psi - 1}{\psi}} \right)^{\frac{\gamma}{\psi - 1}} \)

(with \( \bar{c} \) representing the trade partner for country \( c \), that is \( \bar{c} \neq c \)), and where \( I_c(A) \equiv \int_{S} \left[ (1 - \gamma) [g_c(k, b, \varepsilon; A) - (1 - \delta)k] + \gamma [k_0 - (1 - \delta)k] \right] \mu_c(d[k \times b \times \varepsilon]), \) for \( c = 1, 2 \)

(x) \( y^{DC}(A) + y^{Xc}(A) = Y_c(A), \) where \( Y_c(A) \equiv \int_{S} y_c(k, b, \varepsilon; A) \mu_c(d[k \times b \times \varepsilon]), \) for \( c = 1, 2 \)

(xi) \( \mu_c'(J, \varepsilon_j) = (1 - \gamma) \left\{ \left( (k, b, \varepsilon_i) \mid (g_c(k, b, \varepsilon_i; A), b_c'(k, b, \varepsilon_i; A)) \in J \right) \right\} \right\} \}

\( \phi_c^e(d[k \times b \times \varepsilon_i]) + \gamma \chi(k_0) M(\varepsilon_j), \)

\( \forall (J, \varepsilon_j) \in S, \) defines \( \Gamma, \) where \( \chi(k_0) = \{ 1 \text{ if } (k_0, 0) \in J; 0 \text{ otherwise} \}, \) for \( c = 1, 2 \)

In closing this section, we define each country’s GDP as the value of its total production, denominated in units of its own final goods. Given the notation in item (x) above, this can be conveniently expressed as: \( GDP_c \equiv \frac{p^{DC}}{p^c} Y_c. \)

5 Calibration

The length of a period in our model corresponds to one year. We assume the household period utility function takes the form:

\[ u(C_i(A), N_i(A)) = \frac{1}{1 - \phi} \left[ \left( C_i(A) - \frac{\kappa}{\eta} N_i(A)^\eta \right)^{1-\phi} - 1 \right], \]

adopting the preferences of Greenwood, Hercowitz and Huffman (1988). Because it eliminates wealth effects on labor supply, this is a common specification in international business cycle models.\(^7\)

\(^7\)See, for example, Devereux, Gregory and Smith (1992), Raffo (2008), and Alessandria, Kaboski and Midrigan (2013). Raffo (2008) shows that its use in a standard two-country real business cycle model can generate the observed countercyclical net flow of goods across countries.
The household discount factor $\beta$ is chosen to deliver a long-run annual real interest rate of 4 percent consistent with the measurement in Gomme, Ravikumar and Rupert (2011). Our relative risk aversion in the household utility function $\phi$ is 1, following Schmitt-Grohé and Uribe (2003). The labor exponent in utility $\eta$ is set to deliver a labor elasticity of 1.7, as in Greenwood, Hercowitz and Huffman (1988). Adopting the estimate by Heathcote and Perri (2002), we set the elasticity of substitution between domestic and imported intermediate goods $\rho$ at 0.9.\(^8\) We follow Cooley and Prescott (1995) in setting labor’s share in production $\nu$ equal to 0.6. The firm liquidation rate $\chi$ is 0.0869, ensuring that our model matches the average exit rate among firms in the Bureau of Labor Statistics’ Business Dynamics Statistics database (BDS) over 1979 - 2007.

We set the capital depreciation rate $\delta$ to imply a long-run aggregate investment-to-capital ratio consistent with that for the average annual private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables, controlling for growth. Given that value, we set capital’s share $\alpha$ in the intermediate-good production function to reproduce the 2.3 average annual private capital-to-GDP ratio over the same period. The weight on labor in utility $\kappa$ is selected so that households work one-third of their time in steady state. We choose the weight on domestic intermediate goods in final-good production $\omega$ to imply a steady state imports-to-GDP ratio at 9 percent, matching the imports of goods and services for the U.S. between 1960Q1 and 2006Q4.

The collateral constraint parameter $\theta_c$ is 0.95 in steady state for $c = 1, 2$. This implies a steady-state aggregate debt-to-asset ratio of 0.31, near the 0.37 average from nonfarm nonfinancial businesses over 1954 - 2006 in the Flow of Funds. We set the initial capital stock for new firms $k_0$ to imply that, in steady state, the employment size of a new firm is 0.285 that of a typical firm, reproducing the average relative employment size of a new firm in the BDS over 1979 - 2007. The persistence and standard deviation of the firm-level productivity process, $\rho_\varepsilon$ and $\sigma_\varepsilon$, are jointly chosen for consistency with two aspects of establishment-level investment rates documented by Cooper and Haltiwanger (2006) using panel data from the Longitudinal Research Database. They report a cross-sectional mean investment-to-capital ratio averaging 0.12, and a standard deviation of investment rates averaging 0.34; examining a sample of firms in our model’s steady state selected for consistency with the Cooper and Haltiwanger sample, we obtain an average $i/k$ at 0.14 and a standard deviation of $i/k$ at 0.43. Resulting parameter values are summarized below in Table 1.

---

\(^8\)Corsetti et al. (2008) estimate the elasticity of substitution between home and foreign tradeables through the lens of a two-country model with tradable and non-tradeable goods, using the U.S. to represent the home country and the trade-weighted aggregate of Canada, Japan and EU-15 as the foreign country; their resulting estimate is 0.85. Given the wide range of estimates of the Armington elasticity in the literature (see Ruhl (2008)), we also report results from a version of our model with $\rho = 1.5$ in section 6.1.3 below.
### TABLE 1. Parameter values

<table>
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<th></th>
<th>β</th>
<th>ϕ</th>
<th>κ</th>
<th>η</th>
<th>ρ</th>
<th>ω</th>
<th>δ</th>
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<td></td>
<td>0.962</td>
<td>1.000</td>
<td>1.480</td>
<td>1.588</td>
<td>0.900</td>
<td>0.930</td>
<td>0.067</td>
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<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>ν</th>
<th>ρₐ</th>
<th>σₐ</th>
<th>θ</th>
<th>γ</th>
<th>k₀</th>
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<tbody>
<tr>
<td></td>
<td>0.345</td>
<td>0.600</td>
<td>0.757</td>
<td>0.026</td>
<td>0.950</td>
<td>0.087</td>
<td>0.304</td>
</tr>
</tbody>
</table>

NOTE.— Preference parameters: β (discount factor), ϕ (relative risk aversion), κ (weight on labor), η (curvature on labor), ρ (Armington elasticity), ω (home bias). Production parameters: δ (capital depreciation rate), α (capital share), ν (labor share), ρₐ and σₐ (persistence and standard deviation of firm-specific productivity shock). Collateral constraint and other parameters: θ (limit on debt per unit cash), γ (exit rate), ξₖ (relative capital of a new firm).

### 6 Results

Throughout this section, we examine a series of impulse response figures (solved as perfect foresight transition paths) to gauge whether our model supports the idea that a large financial shock in the U.S., propagated through goods trade alone, could have caused the recent global recession phenomenon. One limitation of this approach versus a stochastic simulation is the fact that the credit shock we consider is entirely unanticipated. Recall, though that no firm in our economy pays dividends before it is forced to leave the economy by an exit shock, and firms undertake investments as close to efficient levels as their financial situations permit. Given these decision rules, the surprise nature of the shock is likely innocuous. A separate concern is whether perfect foresight of the shock’s remaining path after impact may alter firms’ and households’ actions relative to what would happen under uncertainty. This we cannot dismiss without comparison to a stochastic impulse; given the transitory shock we consider, we expect perfect foresight dampens the domestic contraction somewhat, and thus transmission abroad.

#### 6.1 Credit crisis

We begin by examining the responses in our calibrated model economy to a credit shock confined to country 1. The shock we consider in Figures 4 and 5 is a 70-percent drop in the country 1 borrowing constraint parameter θ₁. We assume θ₁ remains at its low value for three periods and thereafter recovers rapidly; we set the persistence at 0.3, following the calibration exercise in Khan.
and Thomas (2013). Our chosen drop in $\theta_1$ implies that total debt of firms in country 1 declines by roughly 45 percent from peak-to-trough. This is consistent with Ivashina and Scharfstein’s (2009) finding, using Reuters DealScan data on new lending to large corporations, that loans used to fund investment in equipment and structures fell 48 percent between 2007 and 2008.

### 6.1.1 domestic responses

Figure 4 shows the impulse responses in country 1. The credit shock affects cash-poor firms’ current investment decisions and hence their capital stocks for the next period. Because capital stocks are already in place when the shock hits, the responses in aggregate quantity variables are modest in the first date of the shock. Nonetheless, given the increased misallocation that will soon arise from cash-poor firms’ worsened ability to finance levels of investment consistent with their productivities, households immediately foresee a fall in their lifetime wealth and a lower return to investment. This reduces their incentives for saving, matching depressed aggregate investment demand. Thus, the demand for final goods falls, reducing demand for intermediate goods, depressing domestic GDP, wages and employment, as well as the demand for imports.

**FIGURE 4. Credit shock: domestic responses**

![Graphs showing impulse responses in country 1](image)

**NOTE.**– Country 1 responses to an unanticipated drop in the country 1 collateral constraint parameter.

$\theta_1$ is 70 percent below normal for 3 periods and thereafter reverts to normal with persistence 0.3.

---

9 In the sample of advanced economies studied in Reinhart and Rogoff (2009), the average number of banking crises between 1945 and 2008 was 1.4, and the average fraction of years countries spent in crises was 7 percent. These observations imply that the probability that a crisis continues from one year to the next once a crisis has started is 0.3125. We adopt this value for the persistence of our credit shock.
After the first period, the credit shock begins to have more direct effects on firms’ production through their capital stocks. First, the initial decline in aggregate investment implies less capital in the aggregate than usual. Second, and more importantly, that aggregate stock is unusually misallocated. As noted above, tightened collateral constraints have particularly adverse implications for the investments of cash-poor firms with relatively high productivity levels. This explains the fall in measured TFP in the lower right panel of Figure 4, despite the absence of any direct shock to aggregate productivity. As this happens, the declines in aggregate quantities grow more pronounced, particularly during the next three periods when firms’ capital stocks are most affected by tight credit conditions. Over these dates, firms sharply curtail their borrowing and investment. Aggregate debt falls by around 45 percent over the first three periods after the shock, then gradually reverts to its steady state level. Investment falls to 12 percent below average one period after the shock, then gradually recovers. The fall in investment reduces demand for final goods, in turn reducing the demand for domestic and imported intermediate goods. Thus, intermediate goods production drops off, as is reflected in the GDP panel. GDP falls by 2.9 percent after one period, reaching a trough at 3.2 percent below normal two periods later. As with GDP, the response in total hours worked closely tracks endogenous TFP; at its trough three periods after the shock’s impact, the labor input is roughly 2.12 percent below normal. Given these declines, the initial consumption rise is sustained only for one period; it declines thereafter until the GDP trough date, where it is 2.7 percent lower than normal.

As noted above, the credit shock reduces production among the firms supplying domestic intermediate goods. However, because these firms’ capital stocks are predetermined from the period preceding the impact of the shock, the initially small drop in labor supply is insufficient to reduce their production to match the fall in domestic demand for their products. The resulting excess domestic supply is thus exported to country 2, so country 1’s exports temporarily rise. This outflow of country 1 intermediate goods is short-lived, however, as the reduced investments among intermediate goods firms begin to be reflected in their capital stocks over subsequent periods, driving up export prices. Export volumes are 2.3 percent below normal four periods after the shock’s impact, at which point they begin a very gradual recovery. Reduced demand for final goods directly implies reduced demand for imports of foreign intermediate goods. Imports follow path similar to that of endogenous TFP, and trough around 3.6 percent below normal.
6.1.2 responses abroad

Figure 5 displays the impulse responses in country 2 arising from the credit shock in country 1. As mentioned above, country 2 experiences a large influx of intermediate goods from country 1 at the date of the shock. Because the two countries’ intermediate goods are complements in the final-good production function, this temporary rise in country 2 imports raises its demand for its own intermediate goods. This stimulates local production, leading to temporary rises in GDP (0.4 percent) and employment (0.2 percent) at the impact date. The resulting increase in the production of final goods also temporarily raises consumption and investment by 0.3 and 2 percent, respectively. Aggregate debt rises for several periods, since the rise in investment encourages more borrowing among intermediate goods firms; their increased cash holdings accommodate this, as the country 2 limit on debt-to-cash ratios, $\theta_2$, is unchanged.

**FIGURE 5. Credit shock: international transmission**

![Graphs showing impulse responses](image)

NOTE.—Country 2 responses to the country 1 credit shock described in notes to Figure 4.

As country 1’s exports fall below normal, country 2 begins to experience the negative effects of the credit shock in country 1. Note that this has nothing to do with aggregate productivity in country 2. With no change in credit conditions there, the extent of misallocation is unaltered, so measured TFP stays at its normal level.

The reductions in country 1’s production and exports to country 2 soon begin to curtail domestic demand for intermediate goods in country 2. Compounded by country 1’s low demand for imports from country 2, this reduces equilibrium production of intermediate goods and, in turn, final goods. GDP and employment fall, discouraging consumption. Intermediate goods firms’ reduced demands
for investment imply reduced needs for borrowing, so aggregate debt declines; once it falls below its steady state level, it remains there for many periods.

Quantitatively, the overall effects of country 1’s credit shock on country 2 are quite small compared to the observed depth of recessions outside of the U.S. during the recent financial crisis. Because the main sources of international transmission in our model economy are the shortage of export supply and the weak demand for imports in the country directly affected by the shock, the size of international trade calibrated to the U.S. data is insufficient to cause a large recession in its trade partner economy. If the trade share is set to a counterfactually high value (0.80), GDP, consumption and hours worked in country 2 fall by 4.34 percent, 1.72 percent and 2.76 percent, respectively, and they remain low for several periods; however, as in the calibrated exercise shown in Figure 5, these effects are delayed.¹⁰

6.1.3 implications of greater trade openness and substitutability

In this subsection, we investigate how the propagation of credit shocks in our model economy is affected by countries’ openness to trade and by the degree of substitutability in the goods they trade. In particular, we examine two cases. In the first, we consider what happens when home bias is weakened relative to our baseline calibration, so that the average volume of trade is greater. In the second, we compare our baseline model’s results to those obtained when traded goods are more substitutable across countries. Throughout these exercises, we study an AR(1) credit shock reducing country 1’s collateral constraint parameter, \( \theta_1 \), by 70 percent for one period, with gradual recovery thereafter governed by persistence 0.7.

In our baseline calibration, imports are 9 percent of GDP in steady state, matching the U.S. average over 1960Q1 to 2006Q4. However, the import share is significantly higher for most other advanced economies; indeed, as of 2006Q4, the U.S. figure was almost 16 percent. We explore a ‘high trade’ version of our model here by reducing the weight on domestic intermediate goods in final-good production (the home bias parameter \( \omega \)) from 0.93 to 0.82, holding remaining parameters constant; this raises the steady state import share to 20 percent of GDP.

Figures 6 and 7 show the responses to the country-1 AR(1) credit shock in country 1 and country 2, respectively. In each figure, we compare the responses from the baseline calibration to those in the high-trade case. Figure 6 shows that, when countries trade more with each other, the immediate responses in country 1 at the impact of the shock are larger, but the responses in all subsequent periods are dampened relative to the baseline. Recall from above that, when the credit shock hits

¹⁰Figures from this exercise are available on request.
this country, the initial decline in production among domestic firms is comparatively small, given their pre-determined capital stocks. With the far sharper drop in aggregate investment demand, there is excess supply of intermediate goods relative to the needs for domestic final good production, and that extra supply is absorbed by country 2 in equilibrium, temporarily raising country 1 exports. In the high trade version of our model, the initial increase in exports is larger. Given lower home bias, the extent to which the domestic economy can usefully absorb excess production capacity is more limited. Thus, at the shock impact date, more intermediates are sent abroad relative to the baseline case, while domestic firms curtail their employment and investment by more. Given the sharper drop in investment, the demand for final goods falls by more, generating larger declines in GDP and imports relative to the baseline case.

**FIGURE 6. Trade openness: domestic responses**

<table>
<thead>
<tr>
<th>NOTE.</th>
<th>Country 1 impulse responses following shock to country 1 collateral constraint parameter, $\theta_1$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shock reduces $\theta_1$ to 70 percent below its ordinary value for 1 period; thereafter, $\theta_1$ reverts to normal with persistence 0.7. Blue x-curves are responses for the baseline model where $\omega = 0.93$; red o-curves are responses in high-trade model where $\omega = 0.82$.</td>
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</tbody>
</table>

After period 0, the credit shock has more direct effects on the economy, as both the misallocation and the reduced aggregate capital stock arising from the previous period’s investment decisions take effect. Over these dates, the declines in GDP, consumption and employment are dampened with higher international trade. For example, three periods after the onset of the shock ($t = 3$), GDP is 1.45 percent below normal in the baseline model; whereas, it is 1.09 percent below normal in the high trade case. The rate of decline in investment between dates 0 and 1 is also reduced when
the economy is more engaged in international trade, and its subsequent recovery is faster. Given a smaller weight on domestic intermediate goods in the production of its final goods, the country has greater protection against the effects of a shock disrupting its domestic intermediate-good production. Thus, we see more reliance on imports in the lower right panel relative to the baseline case, which helps to sustain consumption in the upper right panel, and both dampens the declines in employment and investment and accelerates their recoveries starting in period 1.

**FIGURE 7. Trade openness: international transmission**

![Figure 7](image)

**NOTE.**—Country 2 impulse responses following shock to country 1 collateral constraint parameter, $\theta_1$.

Shock reduces $\theta_1$ to 70 percent below its ordinary value for one period; thereafter, $\theta_1$ reverts to normal with persistence 0.7. Blue x-curves are responses for the baseline model where $\omega = 0.93$; red o-curves are responses in high-trade model where $\omega = 0.82$.

Figure 7 considers the implications of high trade for international transmission of country 1’s credit shock. At date 0, the initial surge in country 2’s aggregate quantities is amplified by a higher trade share, as the larger temporary rise in country 1 exports induces a greater increase in country 2’s production. In subsequent periods, however, we see that the greater openness to international trade amplifies the transmission of the recessionary effects of the credit shock from country 1. Three periods after the shock ($t = 3$), country 2’s GDP is 0.22 percent below normal, and its investment is down 0.74 percent in the high trade case, versus 0.11 percent and 0.14 percent, respectively, in the baseline model. Since lower home bias implies that a larger share of country 1’s intermediate goods is used in country 2’s production of final goods, country 2 is more exposed to shocks affecting its trading partner. Once the effects of the initial inflow of country 1 intermediate
goods subside, the increased cost of imports from country 1 delivers a larger negative impact on final-good production in country 2, generating larger declines in consumption and investment. As intermediate good firms in country 2 scale back their investments, they require less external finance, so we see a steady decline in aggregate debt over many periods. With the larger contraction in intermediate-good production, employment falls by more. Finally, because the shock-induced gap between country 1’s consumption and that in country 2 is narrowed with greater openness to trade, the appreciation in the real exchange rate is smaller.

We next consider implications of the degree of substitutability between domestic and imported intermediate goods for the responses to the same credit shock as above. In our baseline calibration, the Armington elasticity, \( \rho \), is 0.9, so domestic and imported intermediate goods are complements. An alternative value at 1.5 is often used in international business cycle models.\footnote{See, for example, Backus, Kehoe and Kydland (1994) and Chari, Kehoe and McGrattan (2002).} We adopt this elasticity as a ‘substitutes’ case for comparison with our baseline model’s responses in Figures 8 and 9 below, again holding other parameters fixed.

**FIGURE 8. Traded-good type: domestic responses**

![Figure 8](image-url)

**NOTE.**—Country 1 impulse responses following shock to country 1 collateral constraint parameter, \( \theta_1 \). Shock reduces \( \theta_1 \) to 70 percent below its ordinary value for one period; thereafter, \( \theta_1 \) reverts to normal with persistence 0.7. Blue x-curves are responses for the baseline model where \( \rho = 0.90 \); red o-curves are responses in high substitutability model where \( \rho = 1.5 \).

Beginning with the domestic responses in Figure 8, we see that, once the credit shock begins to have direct effects on firms’ production through their lower capital stocks one period after the shock,
the recessionary effects of the domestic credit shock are marginally greater when traded goods are more substitutable. Three periods after the impact date, country 1’s GDP is 1.64 percent below its steady state level when goods are more substitutable ($\rho = 1.5$), versus 1.45 percent in the baseline case. This is largely driven by a greater reduction in the production of intermediate goods for export (lower left panel). Given the misallocative effects of country 1’s credit shock on its aggregate productivity, the intermediate goods it produces are more expensive in units of country 2 final goods following the shock. When country 2 can more easily substitute away from these goods, exports fall by more, and the overall level of production in country 1 falls slightly further. This is counterbalanced to an extent by country 1’s greater ability to substitute imported goods for its own intermediates and the appreciation of country 1 currency (shown in Figure 9), which mitigate the fall in imports and limits amplification in the consumption and labor responses.

**FIGURE 9. Traded-good type: international transmission**

![Diagram showing international transmission](image)

**NOTE.** Country 2 impulse responses following shock to country 1 collateral constraint parameter, $\theta_1$. Shock reduces $\theta_1$ to 70 percent below its ordinary value for one period; thereafter, $\theta_1$ reverts to normal with persistence 0.7. Blue x-curves are responses for the baseline model where $\rho = 0.90$; red o-curves are responses in high substitutability model where $\rho = 1.5$.

Consistent with our reasoning above, Figure 9 shows international transmission of country 1’s credit shock is much weaker when the two countries’ intermediate goods are more substitutable. At $t = 3$, GDP in country 2 is only 0.01 percent below normal, versus 0.11 percent in the baseline model. In the substitutes case, the initial inflow of goods from country 1 induces a smaller initial increase in country 2’s output, as the rise crowds out some of its own intermediate goods production.
As such, the initial jumps in consumption, investment, and employment are all muted. Conversely, once country 1’s aggregate capital and endogenous productivity begin falling, higher substitutability shields country 2 from the recessionary effects originating in country 1. Final good production in country 2 declines very little, dampening the movements in investment and consumption. This result is consistent with Heathcote and Perri’s (2002) finding that, in a two-country business cycle model driven by country-specific productivity shocks, the cross-country correlation of GDP falls with the elasticity of cross-country substitution under complete international financial markets.

In closing this section of results, it is worth noting that the assumption of complete international financial markets has little implication for the transmission of our credit shock. When we modify the baseline model allowing households access to only risk-free non-contingent bonds issued by each country, results are virtually unchanged. Three periods after the credit shock hits country 1, GDP in country 2 is 0.11 percent below normal under the incomplete market assumption, precisely the value in our baseline model. This is reminiscent of the finding by Heathcote and Perri (2002) that equilibrium allocations in the bond-economy model are very similar to those in the complete-markets model, regardless of the elasticity of substitution between country 1 goods and country 2 goods, the degree of cross-country spillover in productivity shocks, and the persistence of the shocks (assuming they are stationary). Kehoe and Perri (2002) also show that impulse responses following a country-specific productivity shock (without exogenous spillovers) are very similar in an incomplete markets version of the IRBC model to those with complete financial markets.

### 6.2 Productivity shock

How does the propagation of credit shocks in our model economy compare to the dynamic responses following country-specific productivity shocks? Here, we examine the results following an exogenous TFP shock to country 1. To control the comparison, we choose the size and persistence of the productivity shock to emulate the baseline model’s impulse response of measured TFP in country 1 following the AR(1) credit shock in Figure 6. This leads us to set the initial drop in country 1’s exogenous TFP at 1.4 percent, and to assume the series recovers with persistence 0.6. The results of this exercise at home and abroad are shown in Figures 10 and 11; Tables 2 and 3 compare these outcomes to those following the credit shock discussed above.

In Figure 10, we see that the drop in country 1’s exogenous TFP reduces its production of intermediate goods and hence GDP immediately, leading to a fall in employment. The contraction in the supply of domestic intermediate goods leads to lower exports and below-average final-good

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12 Figures from this exercise are available on request.
production. The latter implies reduced demand for imports of country 2 intermediates, alongside a fall in domestic consumption and investment.

**FIGURE 10. Productivity shock: domestic responses**

![Graphs](image)

**NOTE.**– Country 1 impulse responses following exogenous TFP shock to country 1. Shock selected to match the (baseline model) path of country 1 measured TFP in exercises above where $\theta_1$ falls by 70 percent for one period and reverts to normal with persistence 0.7. Resulting shock is a 1.4 percent drop in $z_1$ for one period, followed by steady-state reversion with persistence 0.6.

Despite our model’s financial frictions, a TFP shock has a very even incidence across country-1 firms (unlike a credit shock). Thus, as in the closed-economy setting of Khan and Thomas (2013), the distribution of production is largely unaffected, implying no change in the extent of misallocation and no endogenous unraveling of TFP. For that reason, we see no subsequent declines in domestic quantity variables, in contrast to the results following the credit shock in Figure 6; results here are similar to those in the model of Kehoe and Perri (2002) with complete financial markets.

**TABLE 2. Domestic peak-to-trough declines**

<table>
<thead>
<tr>
<th></th>
<th>TFP</th>
<th>GDP</th>
<th>Cons.</th>
<th>Invest.</th>
<th>Labor</th>
<th>Debt</th>
<th>Exports</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>credit shock</td>
<td>1.39</td>
<td>2.77</td>
<td>1.39</td>
<td>9.49</td>
<td>1.75</td>
<td>44.57</td>
<td>1.22</td>
<td>3.07</td>
</tr>
<tr>
<td>TFP shock</td>
<td>1.40</td>
<td>2.15</td>
<td>1.26</td>
<td>6.49</td>
<td>1.36</td>
<td>0.91</td>
<td>0.62</td>
<td>2.39</td>
</tr>
</tbody>
</table>

**NOTE.**– Maximum declines in country 1 series in response to domestic shocks. Credit shock (row 1) is 70 percent decline in country 1 collateral constraint parameter $\theta_1$ with persistence 0.7. TFP shock (row 2) is a 1.4 percent fall in exogenous productivity, with persistence 0.6.
In Table 2, we compare the depth of the trough of aggregate variables in country 1, as percentage deviation from their respective steady state levels, in response to the credit shock and the productivity shock. Although the two shocks reduce measured TFP by the same amount, notice that the credit shock generates larger declines in all of the other domestic aggregates. The differences with respect to investment are substantial; those with respect to debt are dramatic.

As noted above, a credit shock disproportionately hinders the investment activities of firms with low cash on hand, distorting the allocation of capital further from the efficient one. This shock has a particularly sharp impact on domestic investment, as the households that own firms anticipate low rates of return over coming periods. The fall in investment partly explains the decline in debt. However, it is directly compounded by tight credit, given both the drop in $\theta_1$ and the endogenous reductions in firms’ cash that generates. By contrast, a productivity shock only affects firms’ borrowing ability through their effects on firms’ static profits; there, the decline in debt arises only from a decline in investment demand and is thus minor.

The credit shock also generates much larger adverse effects on exports and imports, relative to the TFP shock. Imports fall 1.28 times as far under the credit shock than happens in response to the TFP shock; the drop in exports almost doubles (1.98) as we look from the TFP shock row to the credit shock row. These results are consistent with empirical findings that financial constraints exacerbated the sharp decline in international trade during the U.S. financial crisis (see, for example, Behrens, Corcos and Mion (2013) and Coulibaly, Sapienza and Zlate (2011)).

FIGURE 11. Productivity shock: international transmission

NOTE.– Country 2 responses following TFP shock to country 1 described in notes to Figure 10.

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Bems, Johnson and Yi (2013) conclude that credit shocks account for about 15-20 percent of the great trade collapse of 2008-2009.
Figure 11 displays the impulse responses in country 2 arising from the TFP shock taking place in country 1. With the drop in country 1’s demand for imports from country 2, alongside a reduced inflow of intermediate goods from country 1, intermediate goods firms in country 2 cut back their production and hiring at the impact of the shock. Given international risk sharing and the absence of trade barriers in our model, investment rises briefly as households in country 1 redirect their savings in response to the rise in country 2’s relative productivity. This rise in investment induces a temporary increase in borrowing. Thereafter, as reduced international goods trade continues to discourage production in country 2, its investment falls, reaching below-average levels after two periods, and debt begins to decline correspondingly. Meanwhile, consumption falls gradually.

<table>
<thead>
<tr>
<th>TABLE 3. Peak-to-trough declines abroad</th>
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<tbody>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>credit shock</td>
</tr>
<tr>
<td>TFP shock</td>
</tr>
</tbody>
</table>

NOTE.—Maximum declines in country 2 series in response to country 1 shocks described in notes to Table 2.

Finally, Table 3 compares international transmission of credit versus productivity shocks, measuring the peak-to-trough declines in country 2 following each country 1 shock. Given its greater impact effects on exports and imports above in Table 2, we see here that the credit shock delivers far greater transmission than does the productivity shock. Aggregate effects abroad under the credit shock are roughly twice the size arising from the TFP shock.

7 Concluding remarks

Our goal in this paper was to explore the extent to which a large credit shock in one country is transmitted to its trade partners through goods trade alone. To that end, we developed a two-country equilibrium business cycle model wherein the producers of intermediate goods used in final production at home and abroad face persistent idiosyncratic productivity shocks and collateral constraints limiting the sizes of their investment loans. We calibrated our model symmetrically using conventional long-run aggregate moments drawn from postwar U.S. data, alongside the U.S. average share of imports in GDP and average debt-to-asset ratio, as well as a series of micro-level moments drawn from the Business Dynamics Database and Longitudinal Research Database.

Our model predicts that a credit shock in one country yields a sharp contraction there and a delayed downturn in the economy of its trade partner. When a country’s credit availability tightens,
the domestic allocation of production is distorted, as an increased number of cash-poor firms find it harder to finance investment consistent with their productivity levels. Domestic investment is discouraged, depressing the demand for final goods, and thus the demand for intermediates. Demand for imported intermediates falls persistently, as these goods complement domestic intermediates in the production of final goods. While exports briefly rise with the release of excess domestic intermediates, this is soon reversed. There forward, the country’s trading partner begins to experience real negative consequences. However, the economic damage abroad is small, so long as we confine attention to calibrations consistent with the size of international goods trade in postwar U.S. data.

International transmission is greater when countries are more open to trade and the goods they trade are less substitutable, as each country is more exposed to the health of its trading partner. While the transmission of a credit shock is minor in our calibrated model, we show it is nearly double that of an aggregate TFP shock carefully selected for comparability, while still driven entirely by trade volumes. Given the disproportionate effects of credit shocks on investment in our model, they generate reductions in international trade substantially larger than those caused by productivity shocks. As such, our framework may be useful in interpreting recent empirical evidence suggesting that financial constraints contributed to the collapse in international trade after the start of the U.S. financial crisis. Still, under reasonable parameterizations, the powerful propagation effects of financial shocks uncovered in closed economy settings with the multi-dimensional firm heterogeneity we have included here do not in themselves extend across borders in an otherwise standard international business cycle framework without financial contagion.

Our model offers a rich and plausible microeconomic setting with financial frictions disparately affecting heterogenous firms on an IRBC stage. We have abstracted from presumably important international financial linkages here to focus squarely on the question of whether an isolated financial shock transmitted through goods trade alone can explain much of the recent global recession. Our results suggest not. As discussed above, a key mechanism propagating the credit crisis in our setting is an endogenous decline in aggregate productivity for the country directly affected by tight credit, which arises from capital misallocation. A financial linkage simultaneously worsening the allocation of production abroad would almost certainly imply strong comovement in our model. One natural, if cumbersome, extension that could deliver this would be one wherein firms carry two financial state variables, with firms borrowing at home and abroad subject to distinct collateral constraints associated with each financial source. There we would expect to see stronger cross-country spillovers, highlighting the importance of international real-financial linkages.
References


