Partial adjustment without apology

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Abstract

Many kinds of economic behavior appear to be governed by discrete and occasional individual choices. Despite this, econometric partial adjustment models perform relatively well at the aggregate level. Analyzing the classic employment adjustment problem, we show how discrete and occasional microeconomic adjustment is well described by a new form of partial adjustment model that aggregates the actions of a large number of heterogeneous producers facing fixed costs of factor adjustment. In the market equilibrium of this model, employment responses to aggregate disturbances include changes both in a target employment selected by establishments and in the measure of establishments actively adjusting to this target. Yet the model retains a partial adjustment flavor in its aggregate responses.

Previous research involving discrete factor adjustment has been conducted almost exclusively under the assumption of exogenous prices, given the complications presented by nontrivial heterogeneity in production. We demonstrate how such complications can be limited, allowing both general equilibrium analysis and the convenience of linear solution methods. We also show how our framework is easily generalized to accomodate persistent idiosyncratic shocks. This generalization allows both greater consistency with the microeconomic dynamics of factor adjustment, as well as application to a much broader set of questions involving discrete individual choices, within a tractable equilibrium model.

1 Introduction

In many contexts, actual factor demands clearly involve complicated dynamic elements absent in static demand theory. For example, empirical studies of the market demand for labor typically find that lags, either of demand or of the determinants of demand, contribute substantially to the explanation of employment determination. The most frequent rationalization of such lags is that individual plants face marginal costs that are increasing in the extent of adjustment, leading them to choose *partial adjustment* toward the levels suggested by static demand theory. Many empirical studies also indicate, however, that the partial adjustment model is inconsistent with the behavior of individual plants or firms. For example, Hamermesh (1989) shows that individual plants undertake discrete and occasional workforce adjustments rather than the smooth changes implied by partial adjustment. Nonetheless, the model continues to be a vehicle for applied work, essentially because it is a tractable way of capturing some important dynamic aspects of market demand. It is frequently thus employed in an apologetic manner, with the researcher suggesting that it is a description of market, rather than individual, factor demand.[1]

We present a *generalized partial adjustment* model in which individual production units adjust in a discrete and occasional manner, yet there is smooth adjustment at the aggregate level. Specifically, individual units face differing fixed costs of adjustment, so the timing of their adjustments is infrequent and asynchronized while aggregation across plants leads to a smooth pattern of aggregate factor demand well-approximated by the standard partial adjustment model. Our exposition of this model's relation to the traditional model commonly used in empirical work is unique to this paper.

Our basic framework is sufficiently tractable that it has already been applied to examine several topics, among them price adjustment and capital investment.[2] Here, we apply it to employment which, relative to the investment application, requires a different timing to trace the resulting distribution of production. We provide the first comprehensive presentation of the framework so that researchers may conveniently adapt it to study other problems. To facilitate its broad application, we then extend the method to allow for persistent idiosyncratic shocks.

Our model provides a microeconomic foundation for the variety of plant-level adjustment examined in the empirical work of Caballero and Engel (1992, 1993) and Caballero,

[1] See, for example, Kollintzas (1985).

[2] Dotsey, King and Wolman (1999) use the framework to analyze price adjustment; Thomas (2002) uses it to examine investment.

Engel, and Haltiwanger (1997). There, individual production units are assumed to adjust employment probabilistically, with adjustment probabilities being a function the difference between a target level of employment and actual employment. Aggregating from such adjustment hazard functions, which are their basic unit of analysis, they examine the implications of the resulting state-dependent adjustment behavior for aggregate employment demand dynamics. In the absence of a microeconomic foundation for such probabilistic adjustment, Caballero and Engel (1993, p. 360, paragraph 2) explain that they "trade some deep parameters for empirical richness." In contrast, we explicitly model the plant's adjustment decision as a generalized (S, s) problem and derive the adjustment hazard functions that are the starting point of previous research.[3]

One key stylized fact uncovered in the empirical literature is that an important route through which aggregate shocks affect aggregate employment is by changing the fraction of plants that choose to adjust. Accordingly, we develop a model where the aggregate adjustment rate is an endogenous function of the state of the economy. While our generalized model is not observationally equivalent to the traditional partial adjustment model with time-invariant aggregate adjustment rates, impulse responses establish that it retains the basic features of gradual partial adjustment. Another distinguishing feature of our theoretical approach is that it is convenient to undertake generalized (S, s) analysis in a general equilibrium environment, so that the influence of aggregate shocks on equilibrium adjustment patterns may be systematically studied. Finally, our approach is sufficiently tractable so as to accommodate additional sources of heterogeneity. Thus, beyond achieving consistency with the stylized facts highlighted here, it naturally extends to allow for the richer heterogeneity of actions that is essential in matching other aspects of the microeconomic data on factor adjustment. Moreover, this generalization to include discrete individual adjustments alongside persistent idiosyncratic elements makes our framework amenable to a broader set of applications beyond those considered thus far.

The organization of this discussion is as follows. Section 2 briefly reviews the essential properties of the standard partial adjustment model, and section 3 describes the evidence on microeconomic adjustment patterns that the standard model fails to explain. Next, section 4 develops a model that is consistent with the observation that individual establishments hire varying amounts of labor in discrete and occasional episodes, and it illustrates a resulting hedging effect on the demand for labor. Our model makes the timing of discrete individual employment changes endogenous by assuming that plants face fixed costs of

[3] Generalized (S, s) models were first studied by Caballero and Engel (1999) to explain the observed lumpiness of plant-level investment demand.

adjustment that are random across both time and plants. Establishments respond to this by adopting generalized (S, s) decision rules with respect to labor. At the same time, the framework is readily embedded within a fully specified general equilibrium macroeconomic model, which allows us to examine the influence of deep parameters on the adjustment process. Moreover, with a large number of plants, the model is similar to the traditional partial adjustment model in that it yields a smooth market labor demand. We illustrate several properties of our *generalized partial adjustment model* in section 5 through a series of numerical examples.[4] Beyond its consistency with the evidence on employment adjustment in section 3, the model is also distinguished by its potential to reproduce the sharp changes in market employment demand found in the data during episodes involving large changes in productivity.[5] Moving from our market demand examples, we provide counterpart results that illustrate the role of equilibrium in shaping the aggregate response to shocks. Finally, section 6 demonstrates how our framework is extended to accommodate persistent differences in productivity across establishments, and section 7 concludes.

2 Standard partial adjustment

The standard partial adjustment model relates current employment, N_t, to *target* or desired employment, N_t^*, through $N_t - N_{t-1} = \kappa[N_t^* - N_{t-1}]$, where $\kappa \in (0, 1)$ is the fraction of the gap closed in the period. This specification implies the influence of past actual or desired employment on current employment,

$$N_t = \kappa N_t^* + (1 - \kappa)N_{t-1} = \kappa \sum_{j=0}^{\infty} (1 - \kappa)^j N_{t-j}^*. \tag{1}$$

As shown by Sargent (1978), this empirical partial adjustment model may be derived as the solution to a firm's dynamic profit maximization problem under the assumption that there

[4] Our model is distinguished from earlier generalized cost of adjustment models, as summarized, extended and critiqued in Mortensen (1973), in that it suggests very different dynamics at the establishment-level. Nonetheless, because our model is essentially one with many dynamically related factor demands, it is capable of generating some of the aggregate dynamics that motivated researchers in this earlier area. For example, under unrestricted parameters, interrelated factor demand models were found to be consistent with oscillatory approaches to the long-run position. Our model can also generate such rich dynamics, although it does not do so under the parameters selected here.

[5] This is because the economywide rate of adjustment implied by our model varies with aggregate conditions. The traditional model under-predicts employment changes during such episodes precisely because the adjustment rate there is constant.

are quadratic costs of adjusting the workforce. In the absence of costly adjustment, assume that the firm's workforce declines at the rate $d \in [0,1)$ due to quits or mismatches. If e_t employees are hired at time t, then $N_t = (1-d)\,N_{t-1} + e_t$, and the cost of the workforce adjustment is $\Xi\,(e_t) = \frac{B}{2}e_t^2$, where $B > 0$.[6] Let z_t reflect current productivity, and w_t be the real wage, (both serially correlated random variables known at date t), and let production be $f(N_t, z_t)$. Discounting future earnings by $\beta \in (0,1)$, the firm chooses $\{N_t, e_t\}_{t=0}^{\infty}$ to maximize its expected present discounted value, $\mathbf{E}\left[\sum_{t=0}^{\infty} \beta^t \left(f(N_t, z_t) - \Xi\,(e_t) - w_t N_t\right) \mid z_0, w_0\right]$, subject to $N_t = (1-d)\,N_{t-1} + e_t$ and given initial employment N_{-1}. If the production function is quadratic in employment, it is straightforward to show that

$$N_t^* = [E_t \sum_{j=0}^{\infty} (\beta/\kappa)^j (\chi_a a_{t+j} - \chi_w w_{t+j})], \tag{2}$$

demonstrating that the presence of lags in employment implies leads under rational expectations, as stressed by Sargent (1978).[7]

The key implications of the model are: (i) current employment, N_t, is directly related to lagged employment, N_{t-1}, because adjustments are costly, and (ii) expectations of future wages and productivity influence current employment through the target, N_t^*, because, given adjustment costs, its choice will in part determine future employment. Taken together, these features imply that adjustment costs dampen the response to changes in current wage and productivity and yield smooth, gradual changes in employment over time.

3 Disconcerting evidence

While the traditional partial adjustment model offers a tractable framework with which to study gradual aggregate labor adjustment, there is considerable empirical evidence to suggest that the model is *not* consistent with the behavior of individual production units. This evidence suggests a number of stylized facts about individual and aggregate adjustment that we summarize here.

Stylized fact 1: Adjustment at the plant level is discrete, occasional and asynchronous. Hamermesh (1989) examines monthly data on output and employment between 1983 and 1987 across seven manufacturing plants. For each plant, output fluctuates substantially

[6] This captures the idea that the firm's marginal adjustment cost is rising in the extent of employment adjustment; the same idea is incorporated in alternative adjustment cost functions used in applied work.

[7] In (1) and (2), the parameters κ, χ_a and χ_w depend on the adjustment cost parameter B, the discount factor β and the parameters of the production function.

over the sample. Employment exhibits long periods of constancy broken by infrequent and large jumps at times roughly coinciding with the largest output fluctuations. Hence, the plant data are not consistent with the smooth employment adjustment that would arise from convex adjustment costs.

Stylized fact 2: Aggregates exhibit smooth and partial adjustment. Hamermesh (1989) also examines the behavior of the aggregate of his seven manufacturing plants. He finds that fluctuations in aggregate employment resemble the dynamics of aggregate output and appear consistent with smooth adjustment behavior of aggregates. More specifically, he argues that the standard partial adjustment model works quite well at the aggregate level, even though it does not describe the behavior of individual production units.[8]

Stylized fact 3: Adjustment hazards depend on aggregate conditions. Following the econometric literature on discrete choices, an *adjustment hazard* typically refers to the probability that a production unit will undertake a discrete change, with this probability depending on the position of the unit's state variable relative to some target value. Caballero and Engel (1993) construct a general framework for studying aggregate employment changes that can incorporate a variety of assumptions about how adjustment hazards are related to aggregate conditions. Using U.S. manufacturing data from 1961 through 1983, they examine the dynamics of aggregate employment changes under two alternative specifications for the hazard function: (1) a benchmark case with a time-invariant, flat hazard, which corresponds to the traditional partial adjustment model (as shown by Rotemberg (1987)) and (2) an alternative hazard model involving higher moments of the cross-sectional distribution of firms' 'disequilibrium' levels, reflecting state-dependent adjustment behavior. They find large increases in explanatory power for aggregate employment changes in moving from the constant hazard model to a generalized hazard structure and attribute this to the effects of large aggregate shocks upon the employment hazard.

[8]Hamermesh compares log likelihood values from the estimation of a partial adjustment model based on quadratic costs to those from a lumpy adjustment fixed-cost alternative. For plant-level data, the latter achieves much larger likelihood values. Further, his switching model estimates of the percentage 'disequilibrium' required to induce adjustment are large, suggesting that plants vary employment with a non-marginal adjustment only in the presence of substantial shocks to expected output. However, differences at the aggregate level are too small to discriminate between models, as is the case when they are compared using 4-digit SIC data. Thus, lumpy adjustment behavior at the microeconomic level is obscured by aggregation. From this and similar evidence, Hamermesh and Pfann (1996, page 1274) conclude that "observing smooth adjustment based on data describing industries or higher aggregates over time is uninformative about firms' structures of adjustment costs and in no way disproves the existence of lumpy costs."

Stylized fact 4: Adjustment hazards depend on measures of 'micro gaps'. More direct evidence on the importance of state-dependent adjustment hazards is provided by Caballero, Engel and Haltiwanger (1997). Studying the direct relationship between the adjustment hazard at the level of the individual production unit and the extent of that unit's gap between current employment and a measure of desired employment, these authors show that the adjustment hazard depends on the size of this discrepancy. They suggest that individual units may face differential adjustment costs, so that the distribution of adjustment costs governs the adjustment hazard.

Stylized fact 5: Aggregate shocks are much more important in accounting for aggregate responses than are shifts in cost distributions. The empirical analysis of Caballero, Engel and Haltiwanger (1997) also suggests that changes in the distribution of adjustment costs are not central in explaining stylized fact 3. Rather, aggregate shocks induce changes in hazards that are important for aggregates because they produce movements along the micro-distribution of employment imbalances.

4 Generalized partial adjustment

A number of recent theoretical and empirical studies – notably those of Caballero and Engel (1993) and Caballero, Engel and Haltiwanger (1997) – have argued for a richer vision of the adjustment process that can generate the stylized facts discussed above. The framework we develop exemplifies such a model. In particular, it delivers the implication that, while individual establishments' employment adjustments are discrete, (fact 1), their asynchronous timing implies a smooth aggregate employment series similar to that implied by the traditional partial adjustment model, (fact 2). Moreover, an individual production unit's probability of adjustment depends on a measure of the 'gap' between its current employment and a notion of desired employment, (fact 4), and the model can produce substantial responses of employment to aggregate shocks without relying on any shifts in the distribution of adjustment costs, (fact 5). At the same time, our approach is readily incorporated into a general equilibrium model, so that the relationship between adjustment hazards and macroeconomic conditions can be formally established (fact 3).

We assume a large and fixed number of production units, each making discrete choices about their employment adjustment over time. Production at the plant is constant returns in labor and a fixed input, which we normalize to 1, $f(n_t, 1, z_t)$.[9] Any unit that does not

[9] The presence of the fixed input allows determination of the employment choice at the production unit.

adjust its workforce sees it decay at rate d,

$$n_t = (1-d)n_{t-1} + e_t, \tag{3}$$

where e_t represents an active adjustment. We endogenize the timing of such adjustments by introducing fixed costs that are stochastic across units, an approach adopted by Caballero and Engel (1999) in their study of manufacturing investment.[10] Within each date, any individual plant faces a random cost ξ that it must pay in order to adjust its employment prior to current production. This cost is drawn from a time-invariant distribution over $[0, B]$ that is summarized by the CDF $G(\xi)$ and associated density $g(\xi)$.

In the discussion that follows, we integrate our generalized partial adjustment approach into a general equilibrium setting by imposing two restrictions on the prices faced by establishments within each date. First, asset-market clearing will require that all establishments discount their future profit flows by households' marginal rate of substitution between current and future consumption, denoted here by $\beta\frac{p_{t+1}}{p_t}$. Equivalently, establishments value their current output by p_t, the current marginal utility of consumption, and discount their future values by the household subjective discount factor β. Next, the equilibrium wage, w_t, will equal households' marginal rate of substitution between current leisure and consumption, $\frac{D_2u(c,1-N)}{D_1u(c,1-N)}$. Provided that these restrictions are satisfied, the role of households in the economy is effectively subsumed, and equilibrium allocations are retrieved as the aggregate of establishments' decisions.[11]

4.1 Adjustment probabilities

At the start of any date t, each production unit may be identified as a member of a particular time-since-adjustment group, j, where j is the number of periods that have elapsed since the unit's last active employment adjustment. Let n_{jt} represent the start-of-period labor stock associated with a member of time-since-adjustment group j, and let α_{jt} be a shorthand representing any such establishment's probability of a current adjustment, as perceived after the observation of the aggregate state but prior to the realization of

[10] The generalized adjustment model developed here has been used in several general equilibrium applications. Dotsey, King and Wolman (1999) study the dynamics of price adjustment, while Thomas (2002) and Khan and Thomas (2003) investigate investment dynamics and Khan and Thomas (2004) use a similar approach to study (S,s) inventory accumulation. In this study, we use linear approximation methods in the tradition of Sargent (1978) to explore the general equilibrium dynamics, as do Dotsey, King and Wolman (1999) and Thomas (2002).

[11] See Khan and Thomas (2003) for further explanation.

adjustment costs.[12] We limit the number of time-since-adjustment groups by restricting the parameters of our model to ensure that there is some maximum nonadjustment horizon, J, the number of periods within which all production units will adjust their employment with probability 1: $\alpha_J = 1$. This *finite memory* feature is useful in limiting the size of the aggregate state vector, and will be discussed further below.

Let S_t denote the aggregate state of the economy determining prices and expectations. We use the notation $V_j(n_{jt}, S_t)$ to represent the production-time value of a plant that last adjusted employment j periods ago and is entering current production with no change to its start-of-period workforce n_{jt}. Next, we use $V_0(S_t)$ to denote the production-time value of a plant that has paid its current fixed cost in order to adjust its employment prior to production. Based on a comparison of these values, it is straightforward to characterize the discrete adjustment decisions made by individual production units. Given its start of period employment, n_{jt}, and the aggregate state, S_t, an establishment will actively adjust its workforce if its current fixed cost, ξ, does not exceed the value of undertaking the adjustment, that is, if $V_0(S_t) - V_j(n_{jt}, S_t) \geq \xi$.[13]

Because there is a large number of production units within each different time-since-adjustment group, each group is characterized by a marginal plant that finds it just worthwhile to adjust. This marginal plant is associated with a threshold cost $\overline{\xi}_{jt}$ such that

$$\overline{\xi}_{jt} = V_0(S_t) - V_j(n_{jt}, S_t). \tag{4}$$

All production units in the j^{th} time-since-adjustment group with adjustment costs at or below the threshold in (4) will choose to adjust. As a result, the fraction of plants adjusting out of any particular group j, $j = 1, \ldots, J-1$, is given by

$$\alpha_{jt} = G(\overline{\xi}_{jt}). \tag{5}$$

From (4), note that these adjustment fractions are functions of the plant-level state vector, (n_{jt}, S_t). We assume that the stochastic process for productivity, alongside the parameters associated with production, fixed cost draws and household utility, are such that $B < V_0(S_t) - V_J(n_{Jt}, S_t)$ for all relevant values of the vector (n_{Jt}, S_t). This assumption follows naturally from $B < \infty$ and ensures that $\alpha_J = 1$.

[12] Except where necessary for clarity, we suppress commas in subscripts throughout this text.

[13] As will be made explicit below, S_t includes two endogenous vectors that together identify the start-of-period distribution of plants over employments, alongside exogenous aggregate productivity, z_t. We assume z_t follows a Markov process that is taken as given by all agents, as is the evolution of the endogenous aggregate state, A_t, according to a mapping $A_{t+1} = \Psi(A_t, z_t)$ that, in equilibrium, results from the aggregation of individual actions.

4.2 Production-time values

Having described the determination of endogenous adjustment probabilities as functions of the production-time values associated with adjusting and nonadjusting plants, we now state the functional equations that determine these values. We have been explicit above in noting the dependence of prices and adjustment probabilities on the economy's aggregate state. Here, however, we suppress these dependencies to reduce equation length.

The value of a plant that is currently adjusting its labor is

$$V_0(S_t) = \max\Bigg(f(n_{0t}, z_t) - w_t n_{0t} + \beta \mathbf{E}\Bigg[\frac{p_{t+1}}{p_t} \Big[\alpha_{1,t+1} V_0\Big(S_{t+1}\Big) - \xi_{1,t+1} \qquad (6)$$

$$+ (1-\alpha_{1,t+1})\, V_1\Big((1-d)\, n_{0t}, S_{t+1}\Big)\Big] \mid S_t \Bigg]\Bigg),$$

where n_{0t} is freely chosen, and $\alpha_{1,t+1}$ is given by (4) - (5) above.[14] The right-hand side of this Bellman equation involves three expressions. First, there is the flow of current profit. Second, there is the expected discounted value of being a unit that adjusts next period, which occurs with state-dependent probability $\alpha_{1,t+1}$ and implies an expected fixed cost payment, $\xi_{1,t+1}$, conditional on adjustment: $\xi_{1,t+1} = \int_0^{G^{-1}(\alpha_{1,t+1})} xg(dx)$. Finally, there is the value of being a unit that does not adjust next period, an outcome that occurs with probability $(1-\alpha_{1,t+1})$.

For units choosing not to adjust their workforce, there are no further current decisions in this simple model, although there would be in more elaborate settings allowing adjustments on other margins, such as in hours-per-worker. The production-time value of a plant that last adjusted $j = 1, ..., J-2$ periods ago, and is not currently adjusting, is

$$V_j(n_{jt}, S_t) = f(n_{jt}, z_t) - w_t n_{jt} + \beta \mathbf{E}\Bigg[\frac{p_{t+1}}{p_t} \Big[\alpha_{j+1,t+1} V_0\Big(S_{t+1}\Big) - \xi_{j+1,t+1} \qquad (7)$$

$$+ (1-\alpha_{j+1,t+1})\, V_{j+1}\Big((1-d)\, n_{jt}, S_{t+1}\Big)\Big] \mid S_t \Bigg],$$

where $\alpha_{j+1,t+1}$ is given by (4) - (5), and $\xi_{j+1,t+1} = \int_0^{G^{-1}(\alpha_{j+1,t+1})} xg(dx)$. Any such plant produces with its start-of-period labor n_{jt}, then moves to start next period as a member of time-since-adjustment group $j+1$ with workforce $(1-d)n_{jt}$. In other respects, its

[14] As n_{0t} is our model's counterpart to target employment in the traditional model, we occasionally refer to it as n_t^* when making comparisons below.

Bellman equation is analogous to that of current adjustors described above. Finally, the production-time value of a nonadjusting plant that last adjusted $J-1$ periods ago is just as in (7), but reflects the certainty of employment adjustment in the next period:

$$V_{J-1}(n_{J-1,t}, S_t) = f(n_{J-1,t}, z_t) - w_t n_{J-1,t} + \beta \mathbf{E}\left[\frac{p_{t+1}}{p_t}\left[V_0\Big(S_{t+1}\Big) - \xi_{J,t+1}\right] \mid S_t\right]. \quad (8)$$

Before proceeding further, we should offer some comment on the time-since-adjustment subscripts attached to the plant value functions above. So long as adjustment probabilities are optimally chosen, it is clear that the plant-level state is fully captured by (n_{jt}, S_t), and these subscripts are unnecessary. We have chosen to include them here to allow our analysis to accommodate a special case that lies between our generalized model with state-dependent adjustment probabilities and the traditional partial adjustment model characterized by a time-invariant, flat adjustment hazard. This intermediate *time-dependent adjustment* model replaces the endogenous determination of adjustment probabilities according to (4) - (5) with a fixed vector of adjustment probabilities obtained from the deterministic steady state of our state-dependent adjustment model. Thus, it resembles the traditional model in that its adjustment hazard remains fixed over time, but differs in that the hazard is not flat, but rather depends upon a unit's time-since-last-adjustment.[15] When our model is specialized to this case of time-dependent adjustment, j becomes a separate individual state variable determining the probability that a plant will be allowed to adjust its employment, and the subscripts on our value functions are useful in accounting for this.

4.3 Target employment

Adjusting production units exit the j^{th} group for the adjustment group and choose employment so as to maximize the right-hand side of (6), which results in an efficiency condition of the following form:

$$D_1 f(n_{0t}, z_t) - w_t + \beta \mathbf{E}\Big[\frac{p_{t+1}}{p_t}\,(1-\alpha_{1,t+1})\,(1-d)\,D_1 V_1\Big((1-d)n_{0t}, S_{t+1}\Big)\Big|\,S_t\Big] = 0.$$

A notable feature of this condition is that the optimal employment decision on the part of the adjusting production unit is independent of the length of time since it last adjusted and

[15] One may interpret each time-dependent adjustment probability α_j as reflecting the probability that a plant will be able to change its employment at zero fixed cost versus the probability that it will face an infinite adjustment cost.

the size of its workforce at the start of the period, since neither j nor n_{jt} enters into the efficiency condition. This justifies our writing V_0 above in the restricted form that omits these factors. In addition, it implies a common employment adopted by all units adjusting within the same date, and hence the common start-of-period workforce across all members of any particular time-since-adjustment group.

Working with the value function in (7), we can determine the marginal value of additional workers:

$$\begin{aligned} D_1 V_j(n_{jt}, S_t) &= D_1 f(n_{jt}, z_t) - w_t \\ &\quad + \beta \mathbf{E}\Big[\frac{p_{t+1}}{p_t}\,(1-\alpha_{j+1,t+1})\,(1-d)\,D_1 V_{j+1}\Big((1-d)\,n_{jt}, S_{t+1}\Big)\Big|\, S_t\Big]. \end{aligned}$$

These derivatives may be used iteratively to simplify the efficiency condition and derive an alternative implicit expression for the optimal workforce chosen by an adjusting production unit. In particular, n_{0t} solves

$$D_1 f(n_{0t}, z_t) - w_t + \mathbf{E}\sum_{j=1}^{J-1}\Big[\frac{p_{t+j}}{p_t}[\beta(1-d)]^j \varphi_{j,t+j}[D_1 f\left((1-d)^j n_{0t}, z_{t+j}\right) - w_{t+j}]\ |S_t\Big] = 0, \quad (9)$$

where $\varphi_{j,t+j}$ gives the probability that the adjusting unit will make no further adjustment in the next j periods. That is, for $j = 1, ..., J-1$,

$$\varphi_{j,t+j} \equiv \prod_{k=1}^{j}\Big(1 - \alpha_{k,t+k}\Big) = \prod_{k=1}^{j}\Big(1 - G(\bar{\xi}_{k,t+k})\Big). \quad (10)$$

In practice, it is convenient to break the large forward-looking condition determining target employment into J first-order stochastic difference equations. Defining $\Omega_{Jt} \equiv 0$, equation (9) is alternatively written as

$$D_1 f(n_{0t}, z_t) - w_t + \beta(1-d)E\Big[\frac{p_{t+1}}{p_t}\Omega_{1,t+1}\ \mid S_t\Big] = 0, \quad (11)$$

where, for $j = 1, ..., J-1$,

$$\Omega_{jt} \equiv (1-\alpha_{jt})\Big(D_1 f(n_{jt}, z_t) - w_t + \beta(1-d)E\Big[\frac{p_{t+1}}{p_t}\Omega_{j+1,t+1}\ \mid S_t\Big]\Big). \quad (12)$$

Notice that the condition in (9) may be explicitly solved for adjusting units' optimal labor demand in the special case of a Cobb-Douglas production function, $y = zn^{\gamma}$. There,

$$n_{0t} = \left[\frac{E\sum_{j=0}^{J-1}\Big[\frac{p_{t+j}}{p_t}\ \gamma\beta^j(1-d)^{\gamma j}\varphi_{j,t+j} z_{t+j}\Big|\, S_t\Big]}{E\sum_{j=0}^{J-1}\Big[\frac{p_{t+j}}{p_t}\ \beta^j(1-d)^j\varphi_{j,t+j} w_{t+j}\Big|\, S_t\Big]}\right]^{\frac{1}{1-\gamma}},$$

which depends positively on current and expected future productivity and negatively on current and expected future wages. Since this forward-looking labor demand is similar to the behavior of target employment in the standard partial adjustment model of section 2, we sometimes refer to it in this manner. However, it is worth stressing that the economic reasons for this are somewhat different. In the standard model, the firm's labor demand is forward-looking because current adjustments affect the future costs that a firm encounters when it adjusts. Here, by contrast, labor demand is forward-looking because a current adjustor is aware that it may not adjust its employment again in the near future.

The condition in (9) also implies that our generalized adjustment model has a hedging property arising due to forecasted future labor force departures. Assuming the economy's aggregate state is expected to be constant over time, it is straightforward to show that a current adjustor will demand more employment than it would in a frictionless environment. Suppressing changes in wages, interest rates and productivities, the target employment solving (9) is a constant n^*. Let n^s represent the static optimum satisfying $D_1 f(n^s, z) - w = 0$ that would be chosen if the unit could adjust its employment in every period without incurring fixed costs. Given concavity of f, $[D_1 f((1-d)^j n_0, z) - w] < [D_1 f((1-d)^{j+1} n_0, z) - w]$. This implies the summation in (9) evaluated at $n_0 = n^s$ is strictly positive. Moreover, as both this sum and its preceding expression, $D_1 f(n_0, z) - w$, are decreasing in n_0, the dynamic optimum, n^*, must exceed the static optimum.[16]

Production units hire more labor than they currently need to hedge against the possibility that they may face relatively large adjustment costs deterring them from hiring again in the immediate future. Further, n^* will be larger the higher is this probability of future nonadjustment; for instance, given d and $\alpha_2 \cdots \alpha_{J-1}$, a reduced probability of adjustment in the first period after an adjustment, (lower α_1) yields higher values for $\varphi_1, \ldots, \varphi_{J-1}$ and thus a higher value for the summation at any n_0. The higher is the probability that the unit will not restock employment in nearby dates, the stronger is the hedging motive.

4.4 Partial adjustment of market labor demand

The probabilistic approach to microeconomic employment adjustment that we have constructed is consistent with the empirical evidence on rising employment adjustment hazards. Moreover, the framework allows us to aggregate individual plants' labor demand and derive a simple expression for market labor demand. Since the economy is populated

[16] This steady-state exposition of the hedging motive relies on our assumption of a positive exogenous separate rate. Had we assumed that $d = 0$, n^* would, of course, be identical to n^s.

by a large number of production units, we can describe the distribution of plants in any date t using the vector $\theta_t = [\theta_{1t}, ..., \theta_{Jt}]$, with each θ_{jt} representing the fraction of units that begin the period having last adjusted j periods prior to the current date.[17] Letting $\omega_{0t} \equiv \sum_{j=1}^{J} \theta_{jt}\alpha_{jt}$ denote total adjusting units in any date t, the elements of this vector are as follow.[18]

$$\theta_{1t} = \omega_{0,t-1} \tag{13}$$

$$\theta_{jt} = (1 - \alpha_{j-1,t-1})\,\theta_{j-1,t-1} \qquad \text{for } j = 2, ..., J. \tag{14}$$

Market labor demand may then be represented as a moving average of the employment actions of production units, with lag weights determined by adjustment fractions across time-since-adjustment groups. Denoting the target value of employment that solves (9) as n_t^*, we have:

$$N_t = n_t^* \sum_{j=1}^{J} \theta_{jt}\alpha_{jt} + \sum_{j=1}^{J-1} \theta_{jt}\,(1 - \alpha_{jt})\, d^j n_{t-j}^*. \tag{15}$$

This is the third result of our generalized partial adjustment model. The market's dynamic demand for labor describes aggregate employment as a weighted average of past target employments, as in the traditional partial adjustment model (1). Consequently, while the underlying plant-level demands are discrete and occasional, market demand varies smoothly in every period. Further, since each target employment, n_{t-j}^*, $j = 1, \ldots, J-1$, involves expectations of future wages and productivities, so does market labor demand.

While equation (15) shows that our generalized partial adjustment model has a representation similar to the traditional partial adjustment model, there are important differences that eliminate exact aggregate equivalence. In particular, the lag weights here vary over time, because they are composite functions of the adjustment rates α_j, which themselves are functions of plant and aggregate state variables, as consistent with stylized fact 3. Thus, in contrast to the traditional model, our economywide rate of adjustment responds to changes in aggregate conditions, including changes in economic policy.

[17] More precisely, the distribution is completely summarized by the vector θ_t together with a vector of previous target employment levels $[n_{t-1}^*, ..., n_{t-J}^*]$ from which the current support is trivially retrieved. Note that this time-since-adjustment approach allows us to capture the time-varying distribution of establishments over employment levels using a linear systems solution method. We could instead directly track the measure associated with each possible employment level. However, in that case, we would need to include employments that at times have zero population, necessitating a nonlinear solution method as, for instance, in the investment study of Khan and Thomas (2003).

[18] Given a fixed measure of production units, this overall adjustment rate is $\omega_{0t} = 1 - \sum_{j=1}^{J-1} \omega_{0,t-j}\varphi_{jt}$.

4.5 Planning representation

The generalized partial adjustment model described above may be derived as the solution to a single dynamic optimization problem, which makes the link to the standard model of section 2 more direct. We briefly outline this reformulation to illustrate the tractability of the approach and thus its suitability for applications.[19] While we rely on the equivalence between a social planning and competitive equilibrium solution in this section, it is important to stress that the generalized partial adjustment approach can also be applied to settings in which competitive equilibrium is not optimal.[20]

The aggregate representation consolidates the ownership of all plants, differentiated by their time since last adjustment, $j = 1, \ldots, J$, into a single entity, a planner acting to maximize the expected discounted lifetime utility of a representative household. Using the notation $\boldsymbol{\theta}_t \equiv [\theta_{1t}, ..., \theta_{Jt}]$, $\mathbf{n}_t \equiv [n_{1t}, ..., n_{Jt}]$, and $\boldsymbol{\alpha}_t \equiv [\alpha_{1t}, ..., \alpha_{Jt}]$ to describe the economywide distribution of plants, employment, and adjustment fractions across groups, the planner's total available output is:

$$Y_t = f(n_{0t}, z_t) \sum_{j=1}^{J} \theta_{jt}\alpha_{jt} + \sum_{j=1}^{J-1} \theta_{jt}(1 - \alpha_{jt}) f(n_{jt}, z_t). \tag{16}$$

Total employment is an analogous sum of the employments of adjusting and non-adjusting establishments,

$$N_t^D = n_{0t} \sum_{j=1}^{J} \theta_{jt}\alpha_{jt} + \sum_{j=1}^{J-1} \theta_{jt}(1 - \alpha_{jt}) n_{jt}. \tag{17}$$

Finally, economywide adjustment costs are

$$Q_t = \sum_{j=1}^{J} \theta_{jt}\Gamma(\alpha_{jt}), \tag{18}$$

where $\Gamma(\alpha) = \int_0^{G^{-1}(\alpha))} xg(dx)$ is the total volume of costs averaged across plants in a group if fraction α of that group adjusts.

Given the current distribution of plants over time-since-last-adjustment groups, the associated employment levels, and aggregate productivity, the planner chooses fractions

[19] Here, we have chosen to begin our discussion with a description of decentralized actions and now follow with a planning representation. The reverse ordering would have been equally straightforward, which emphasizes the flexibility of the approach. The representation is selected according to its convenience in application.

[20] In its application to the analysis of price adjustment by Dotsey, King and Wolman (1999), for example, the presence of monopolistic competition means that equilibrium is not optimal.

of plants adjusting $(\alpha_{jt})_{j=1}^{J-1}$ and optimal employment for those that are adjusting their workers, n_{0t}, which together determine the next period distribution of plants, $\boldsymbol{\theta}_{t+1}$ and the household's current consumption and work hours. The planner's problem is:

$$\begin{aligned} W(\boldsymbol{\theta}_t, \mathbf{n}_t, z_t) \quad = \quad & \max_{\mathbf{D}_t} \Bigg(u(c_t, 1-N_t) + \beta E W(\boldsymbol{\theta}_{t+1}, \mathbf{n}_{t+1}, z_{t+1} \mid \boldsymbol{\theta}_t, \mathbf{n}_t, z_t) \\ & +\lambda_t [Y_t - Q_t - c_t] \\ & +w_t \lambda_t [N_t - N_t^D], \\ & +s_{0t}\lambda_t \Big[\sum_{j=1}^{J} \theta_{jt}\alpha_{jt} - \theta_{1,t+1} \Big] \\ & + \sum_{j=1}^{J-1} s_{jt}\lambda_t \Big[\theta_{jt}(1-\alpha_{jt}) - \theta_{j+1,t+1} \Big] \Bigg) \end{aligned} \tag{19}$$

subject to $n_{j+1,t+1} = (1-d)n_{jt}$, for $j = 0, \ldots, J-1$, and subject to (16)-(18), where $\mathbf{D}_t = \Big[c_t, N_t, n_{0t}, \{\alpha_{jt}\}_{j=1}^{J-1}, \{\theta_{j+1,t+1}\}_{j=0}^{J-1} \Big]$.

The solution to this problem will satisfy the constraints above with equality and a series of efficiency conditions that follow. First, the standard conditions apply to the choice of household consumption and labor supply,

$$\begin{aligned} \lambda_t &= D_1 u(c_t, 1-N_t) \\ w_t \lambda_t &= D_2 u(c_t, 1-N_t). \end{aligned}$$

From these two equations, it is clear that the output price, p_t, and the real wage, w_t, faced by establishments in the decentralized economy examined above must correspond to the multipliers λ_t and w_t, respectively, if the competitive allocation is to match that obtained here.

Note that the multipliers s_{jt} attached to the distributional constraints in (19) represent date t post-production valuations of establishments that will enter the next date in plant group $j+1$. To clarify the equivalence between the planning allocation and that in the decentralized economy, we define the pre-production valuations of establishments as:

$$v_{jt} \equiv f(n_{jt}, z_t) - w_t n_{jt} + s_{jt}, \text{ for } j = 0, ..., J-1,$$

and we use these, rather than the original multipliers, in representing the optimal adjustment fractions. Efficiency with respect to the choice of α_{jt} requires that the solution to this problem satisfy

$$G^{-1}(\alpha_{jt}) = v_{0t} - v_{jt},$$

so that it is just worthwhile to relocate the marginal plant with cost ξ_{jt} into the adjustment group, and plants with costs greater than this threshold are not adjusted. This determines α_{jt}, $j = 1, \ldots, J-1$, and is equivalent to (4) provided the multipliers v_{jt} attain the same value as before. That this is the case may be seen from the efficiency conditions with respect to $\theta_{j+1,t+1}$, $j = 0, \ldots, J-1$, which imply that the value associated with a plant with employment level n_{jt} satisfies

$$v_{jt} = f(n_{jt}, z_t) - w_t n_{jt} + \beta \mathbf{E}\Big[\frac{\lambda_{t+1}}{\lambda_t}\Big(\alpha_{j+1,t+1} v_{0,t+1} - \Gamma(\alpha_{j+1,t+1}) + (1 - \alpha_{j+1,t+1})\, v_{j+1,t+1}\Big) \mid \boldsymbol{\theta}_t, \mathbf{n}_t, z_t\Big].$$

These expressions are equivalent to the plant Bellman equations of section 4, since the expected adjustment cost conditional on adjustment in (7) is equal to $\Gamma(\alpha_{j+1,t+1})$, the average cost paid by adjusting plants, by definition of $\Gamma(\cdot)$. Finally, the efficiency condition with respect to the choice of n_{0t} may be expressed as (11)-(12), provided $p_t = \lambda_t$ at every date. Therefore, the solution to the planning problem, given the aggregate state $(\boldsymbol{\theta}_t, \mathbf{n}_t, z_t)$, is the same as in the decentralized economy of the previous section.

5 Numerical examples

We use a series of numerical examples to illustrate several interesting properties of the model developed above, and to compare its dynamics to those of the traditional model. We begin with an examination of the model assuming that prices, wages and interest rates are exogenously fixed, as is commonly the case in analyses using the traditional partial adjustment model. Our examples involve functional forms and parameter values that are standard; production at the plant level is described by a Cobb-Douglas production function $f(n, z) = zn^{\nu}$ with $\nu = 0.66$. Total factor productivity has a mean of 1 and follows a first-order autoregressive process with a one-period autocorrelation of 0.9225, roughly consistent with the annual properties of the Solow Residual. The plant's discount factor is $\beta = 0.939$, which corresponds to an annual interest rate of 0.065.[21]

The remaining parameter values are chosen arbitrarily; however, extensive sensitivity analysis has confirmed that the properties of the model we have developed are not highly sensitive to variation in these parameters. First, we assume that the distribution of adjustment costs is uniform with an upper support of 0.008. This yields a distribution of employment across plants that is suitable for illustrating the generalized partial adjustment

[21] These values will be familiar to quantitative researchers; see, for example, King and Rebelo (1999).

model's properties. Next, for the traditional model, we assume the quadratic cost parameter is $B = 4$. This choice facilitates comparison, as it yields a dynamic response that is relatively close to our generalized partial adjustment model with adjustment rates held constant. Finally, we assume a separation rate of $d = 0.06$ and a wage rate of $w = 1.14$.

5.1 The five stylized facts

We have developed a model that is designed to be consistent with stylized facts 1 and 5 of section 3. Specifically, due to fixed costs of adjustment, labor changes at the plant level are discrete and occasional in the model. Moreover, since the distribution of adjustment costs is assumed to be constant over time, it cannot be the source of aggregate fluctuations. Fluctuations must arise through aggregate shocks as suggested by previous empirical work.

The stationary distribution of plants, shown in our first figure, demonstrates the model's ability to reproduce stylized fact 4: adjustment probabilities depend on plants' gaps between actual and target employment. There, we see that adjustment fractions are an increasing function of the time since last adjustment, as the cost of non-adjustment rises with the distance from the target, while the distribution of adjustment costs is identical across groups. Thus, in the second panel of the figure, the distribution of plants across groups is necessarily downward sloping, given the law of motion for $\boldsymbol{\theta}$ in (14).

Figure 2 illustrates stylized fact 2: aggregate employment is characterized by smooth and gradual adjustment. Panels (a) and (b) show percentage deviations in market employment and output from their steady state values, in response to a persistent rise in aggregate productivity, for the three models discussed above. PA corresponds to the traditional partial adjustment model of section 2, where staggered aggregate adjustment arises from the presence of quadratic adjustment costs, while TD represents the response for the generalized model with a fixed vector of time-dependent adjustment fractions corresponding to figure 1. Finally, SD denotes the response in the generalized state-dependent partial adjustment model. There, fixed costs of adjustment dissuade some production units from responding immediately to the rise in productivity. This protracts the aggregate response in employment, and hence output, so that both TD and SD share the humped shape characteristic of the traditional partial adjustment model. This is absent in a frictionless model of employment adjustment, where the shape of the response follows the autoregressive productivity process.

The TD model, with an upward sloping but time-invariant adjustment hazard, matches the traditional partial adjustment model closely. Only at the earliest date of the response does the traditional model move more gradually, due to the rising marginal cost of aggregate employment changes. The size of this initial difference in employment response is nonetheless only about two-thirds of 1 percent. This is in part because plants in the time-dependent adjustment model are not permitted to alter the timing of their employment adjustments in response to shocks, so that all rises in aggregate employment must come from changes in target employment. Moreover, the onset of diminishing returns at the level of the production unit restrains the rise in the employment levels chosen by current adjustors.

While the state-dependent adjustment model shares similar qualitative features with the other staggered adjustment models, the ability of establishments to alter the timing of their employment adjustments at relatively low cost produces two potentially important changes in the market response. First, because aggregate employment is increased through changes in both intensive and extensive margin adjustment, SD produces a substantially larger rise in employment, and hence output, which distinguishes it from the traditional model during the initial dates following the shock. Second, the SD model has the ability to produce more complicated cyclical adjustment patterns; in each panel, its response oscillates slightly above and below that of the traditional model. As neither of these features is present when adjustment rates are held fixed, it is apparent that they arise from changes in adjustment timing at the micro-level.

Figure 3 verifies the importance of the time-varying adjustment hazards by displaying the SD responses in each of the two margins through which aggregate employment is raised. Panel (a) depicts percent changes in extensive margin adjustment through changes in the fraction of production units adjusting, $\omega_{0t} = \sum_{j=1}^{J} \theta_{jt}\alpha_{jt}$, while panel (b) displays intensive margin changes through the employment levels chosen by current adjustors, n_{0t}. Given the persistent nature of the productivity shock, the rewards to early adjustment are expected to be large, thus raising the threshold costs above which adjustment is rejected within each time-since-adjustment group. As a result, adjustment fractions rise across groups, and the number of adjustors in the economy rises 25 percent above its steady state value. This illustrates that stylized fact 3 is reproduced by our generalized partial adjustment model: adjustment rates vary with aggregate conditions. Further, note that the percent rise in target employment per adjusting unit is considerably smaller than that in the number of adjustors. Large increases in individual employments are not worthwhile given decreasing returns in establishment-level production. Thus, in the early dates corresponding to the largest movements in aggregate employment, changes in the numbers of adjusting plants are more important than are changes in the employment chosen by such plants. Moreover, panel (a) demonstrates that it is these extensive margin changes that are responsible for the oscillatory response of the aggregate series in figure 2.

The large rise in the number of adjustors at the impact of the shock causes a large shift in the distribution of production units away from higher time-since-adjustment groups and into group 1 starting the next period. Given upward-sloping adjustment hazards, only a small fraction of these plants adjust again, so many begin the subsequent date in group 2. In this way, the effects of early rises in adjustment rates filter out through subsequent distributions, reducing total adjustment toward trend, then below it once a disproportionate fraction of the population finds its way into time-since-adjustment groups associated with low adjustment fractions. Eventually, the mass of early adjustors works its way sufficiently far out the distribution, where adjustment rates are relatively high, so that total adjustment returns above trend, and the pattern repeats in a dampened fashion.

While illustrative, the response in the total number of adjustors from figure 3 does not fully summarize the effects of changes in adjustment rates on the generalized model's aggregate response, as it fails to reflect the employment levels from which individual units are adjusting. A more complete accounting in provided by figure 4. There, we aggregate the effects of changes in intensive margin versus extensive margin adjustment to provide a decomposition of the market employment response into two underlying components. The first component summarizes n_j*-effects* associated with changes in employment levels across groups (due to changes in target employments). The second summarizes ω_j*-effects* arising from changes in the distribution of plants across these groups at the time of production, $\omega_{jt} \equiv (1-\alpha_{jt})\theta_{jt},\ j=1,...,J,$ (due to changes in the fractions adjusting from each group). Specifically, at each date, the percentage deviation from steady state in aggregate employment is given by

$$\widehat{n}_t = \left[\sum_{j=0}^{J-1}\left(\frac{\omega_j n_j}{n}\right)\widehat{n}_{jt}\right] + \left[\sum_{j=0}^{J-1}\left(\frac{\omega_j n_j}{n}\right)\widehat{\omega}_{jt}\right],$$

where each $\left(\frac{\omega_j n_j}{n}\right)$ reflects the percentage contribution of the j^{th} group to aggregate employment in steady state, and each $\widehat{n}_{jt}$ and $\widehat{\omega}_{jt}$ represent percent deviations from trend in the group j employment and population levels, respectively, at the time of production in date t.

At the onset of the shock, rises in employment associated with current adjustors, n_{0t}, contribute less than half of the percentage rise in the aggregate series. The remainder is due to a rise in the adjustment group, ω_0, associated with this high target and corresponding reductions in the populations of groups associated with lower employment levels. In the following date, adjusting plants again select a high target employment level, and this is compounded by a rise in the employment held by members of group 1, a consequence of the high employment choice of the previous period. These effects of raised targets continue to feed through the distribution, raising the employment levels associated with each subsequent group, for a number of periods. As a result, the n_j component of aggregate employment exhibits the smooth humped shape associated with partial-adjustment. The aggregate series inherits this shape to an extent, but it is both more pronounced in its rise and less smooth in its return to trend, due to the ω_j effects arising from changes in the distribution of plants across groups. High adjustment fractions amplify the aggregate response initially; however, by date 3, when the number of adjustors begins to fall below trend, an increasing fraction of production units operates with relatively low employment levels. This dampens the rise in the aggregate series, then speeds its initial decline, relative to that of the n_j component. Further, just as the disruption in the population distribution produced oscillations in the total adjustors series of figure 3, it also causes overshooting in the ω_j component's response, thereby generating the slight oscillations in the responses of the aggregate series.

We conclude this section with a summary of our main numerical findings thus far. We have seen that our market employment results are consistent with the findings of previous empirical studies (e.g., Caballero and Engel (1993)) in two important respects. First, despite its equivalence to a model with a flat, time-invariant adjustment hazard, the traditional partial adjustment model can approximate the dynamics of our generalized adjustment model relatively well over long samples, since the aggregate series in PA and SD are close over all dates other than in initial episodes following aggregate disturbances. However, second, because the dynamic response of the traditional model tends to be more gradual, its match is poorer in those dates immediately subsequent to aggregate shocks.

Our decomposition of the SD response above explains both findings. First, the similarity between the two models' dynamics over long horizons arises from the presence of a nontrivial adjustment hazard in our model. Although target employment exhibits monotone responses to shocks, individual units' adoption of the target is staggered according to the hazard, which delivers the hump-shaped component underlying our model's market employment responses. This component is characteristic of a traditional partial adjustment response, and it is dominant over most dates. Second, in the early dates following shocks, the traditional model's failure to keep pace with the generalized model does not arise from the flatness of its adjustment hazard, but rather from its time-invariance. In these dates, a second component summarizing the effects of movements in the adjustment hazard becomes important in the SD model. This component amplifies its market employment response relative to both the traditional model and the time-dependent adjustment model, where responses are quite similar despite the fact that the TD hazard is not flat, but upward-sloping. Thus, it is our generalized model's consistency with stylized fact 3 (the dependence of adjustment hazards on aggregate conditions) that distinguishes its employment dynamics relative to both the traditional partial adjustment model and the time-dependent model over such episodes.

5.2 General equilibrium effects

One of the key features of our approach is that discrete micro-level adjustment dynamics can be readily introduced into a general equilibrium setting. So far, we have used the dynamic model to study the influence of variations in productivity on aggregate labor demand and the adjustment decisions of individual units, holding the real wage rate and the real interest rate fixed. The dynamics of a general equilibrium model are more complicated. For example, when a rise in productivity increases labor demand, the resulting wage changes will have implications for both the target employment that adjusting establishments select and the fractions of establishments that choose to adjust from each current employment to this target. In this section, we examine the dynamics of our model in general equilibrium, assuming a particular functional form for the representative household's preferences so as to generate restrictions on the behavior of the wage rate and the interest rate.[22] While our equilibrium analysis is designed to be very simple, it illustrates some important points.

Figure 5 compares the dynamic general equilibrium response to an aggregate productivity shock within the state-dependent generalized adjustment model to the response arising without market-clearing variations in wages and interest rates. Quantitatively, as might be expected, equilibrium price movements sharply dampen the response in employment, and hence output, to a persistent change in productivity. However, in contrast to the investment analysis of Thomas (2002), equilibrium does not eliminate the influence of costly and

[22] In particular, maintaining the functional forms and parameter values assumed above, we assume that the representative household's momentary utility function is $U(C,N) = \log\left(C - \chi\frac{N^{1+\gamma}}{1+\gamma}\right)$, where $\chi = 2.55$ and $\gamma = 0.50$. This specification implies that there is a steady-state level of labor of $n = 0.20$ and a general equilibrium labor supply elasticity of $\gamma^{-1} = 2$ with respect to the real wage rate. Higher values of γ would imply sharper differences between the equilibrium and fixed price models, as these would raise the responsiveness of the wage to changes in employment demand.

discrete adjustment. In particular, the level of employment continues to display a hump in the dynamic response due to these costs, which is also an implication of the standard partial adjustment model discussed in section 2 above.

There are also qualitative changes in both the extensive and intensive margins of employment adjustment with equilibrium movements in wages and interest rates. First, the previous nonmonotonicity in the fraction of units adjusting essentially disappears. This is because equilibrium price changes offset much of the large rise in target employment that would otherwise occur at the impact of the shock. With the rise in target employment dampened, establishments have less incentive to pay fixed costs to move up the timing of their employment adjustments. Thus, equilibrium reduces the jump in the total fraction adjusting, thereby reducing the disruptions to current and future plant distributions that cause these oscillations. Second, the smooth mean reversion in target employment becomes less regular. Nonetheless, target employment continues to be monotonic, and it is the staggered individual responses associated with the model's nontrivial adjustment hazard that produce the hump-shaped aspects of the aggregate quantity responses. More establishments do choose to adjust employment when there is a favorable productivity shock, but they do not all adjust immediately.

In closing our discussion of numerical examples, we revisit the comparison of employment and output responses from figure 2, displaying their general equilibrium counterparts in figure 6. In most respects, the remarks concluding section 5.1 continue to apply. Here, however, the differences across models are slightly less pronounced. First, although general equilibrium dampens responses in all three models, its effect on the state-dependent adjustment model is strongest. This is because equilibrium price changes reduce incentives for the changes in adjustment timing that distinguish SD from the other models. Thus, in the dates associated with the largest aggregate changes, the traditional model now bears a closer resemblance to the generalized model than it did in figure 2. Moreover, given smaller changes in adjustment rates, the distributional effects in our model are now shorter-lived, so its employment and output responses coincide with those of the fixed hazard models more quickly. Thus, the traditional model's ability to proxy for the aggregate response in our richer adjustment model improves somewhat in general equilibrium.

6 Persistent idiosyncratic shocks

To this point, plants have been differentiated only by (i) different realizations of adjustment costs and (ii) different values of the labor stock that they bring into the period.

However, there is ample evidence that establishments are affected by additional persistent plant-level states, such as stochastic variations in productivity. In this section, we demonstrate that our generalized partial adjustment model is tractably extended to allow for persistent idiosyncratic productivity shocks, thus allowing equilibrium analysis in the presence of the richer heterogeneity essential for matching other aspects of the microeconomic data on factor adjustment. Moreover, the same approach can be applied to other persistent exogenous individual states, such as variations in product demand for monopolistic competitors or shifts in the distribution of adjustment costs. Thus, with the extension outlined in this section, our framework can be applied to the many economic settings where discrete choice appears alongside persistent stochastic sources of heterogeneity.

Here, we assume that plant-specific productivity shocks follow an M-state Markov process; $a \in \{a_1, \ldots, a_M\}$ with transition probabilities given by the time-invariant matrix Φ; specifically, the probability of transiting from state a_l to state a_m is given by $\phi(l, m)$, for $l = 1, ..., M$ and $m = 1, ..., M$. This otherwise straightforward extension to our model requires some additional accounting. Thus, we begin by defining notation suitable for describing the joint distribution of establishments over employment and productivity at each date. Next, we show how the aggregation is handled, and then proceed to outline the associated planning problem. For brevity, we omit the corresponding decentralized representation of the economy, although the mapping should be transparent by comparison to section 4 above.

At the start of date t, any establishment is identified by its current productivity draw, a, and its current employment level. We continue to assume that, when not actively adjusted, a plant's employment declines at rate d across dates and that active stock adjustments incur a fixed cost, ξ, drawn from the time-invariant distribution $G(\xi)$. Given the effect of current plant-specific productivity draws on adjustment decisions and on the target employments selected by current adjustors (and hence on future distributions), the economy's aggregate state, S, will now include $M^2 + M$ time-since-adjustment vectors that together describe the current start-of-period distribution of establishments over labor and productivity.

Given this structure, we can use linear approximation to solve our economy if we simply track the distribution of plants according to membership in groups identified by (i) time-since-last adjustment, (ii) productivity draw at the date of last adjustment, and (iii) current productivity draw. As before, employment selected by adjusting establishments does not depend upon the current stock. However, it does depend upon current productivity, given the persistence in this individual state variable.

To study the evolution of plant-level conditions, for each h and each l, we define $\theta_{jt}(h, l)$

as the start-of-date measure of plants that last adjusted j periods in the past to a target employment consistent with a_h, their productivity at the time of the adjustment, and that have current productivity level a_l. Let $\alpha_{jt}(h,l)$ denote the corresponding fractions of each of these groups undertaking active employment adjustment in date t. While adjustment fractions reach 1 within some finite number of periods, the full adjustment horizons for plants now depend upon the productivity they had when they last actively changed their employment and on their current productivity. Let $J(h,l)$ denote the full adjustment horizon associated with plants that had productivity a_h at the time of last adjustment and have current productivity a_l.[23] Because establishments transit across productivities from date to date, each vector $\boldsymbol{\theta}_t(h,l) = [\theta_{jt}(h,l)]$ will have length $J_h \equiv \max\{J(h,1),...J(h,M)\}$. Finally, each of these vectors is associated with the vector of start-of-date employment levels $\mathbf{n}_t(h) = [n_{jt}(h)]$ of length J_h.[24]

6.1 Aggregation

The evolution of the plant distribution may be summarized as follows. First, there are M^2 equations representing the fractions of establishments that are current adjustors and hence will begin the next period with time-since-last adjustment 1. One such equation holds for each current productivity, $l = 1,...,M$, and for each next-period productivity, $m = 1,...,M$. Each represents the fraction of all establishments that have current productivity a_l and adjust from their start of period employment level to the associated target $n_{0t}(l)$, and that will then enter next period identified by $(n_{1,t+1}(l), a_m)$:

$$\theta_{1,t+1}(l,m) = \phi(l,m) \sum_{h=1}^{M} \Big(\sum_{j=1}^{J_h} \theta_{jt}(h,l)\alpha_{jt}(h,l) \Big). \tag{20}$$

Next, there are M^2 sets of equations describing the non-adjusting population. Each set is identified by a particular (past, current) productivity combination, and each contains

[23] As with J in the model above, the horizons here, $J(h,l)$, are endogenous variables recovered in the solution for the economy's steady state. One cost of pursuing a linear systems solution for the dynamics is that we must assume that the economy stays sufficiently local to the steady state that these horizons are impervious to aggregate shocks. To know whether this assumption is reasonable in a given application, one must verify that all endogenous adjustment fractions remain strictly in the $(0,1)$ interval at every date over long simulations.

[24] For example, in the case of a 2-state Markov shock, the start-of-date plant distribution over employment and productivities is summarized by four $\boldsymbol{\theta}$ vectors and two $\mathbf{n}$ vectors: $\boldsymbol{\theta}_t(1,1)$ and $\boldsymbol{\theta}_t(1,2)$, each of length $J_1 \equiv \max\{J(1,1), J(1,2)\}$; $\boldsymbol{\theta}_t(2,1)$ and $\boldsymbol{\theta}_t(2,2)$, each of length $J_2 \equiv \max\{J(2,1), J(2,2)\}$; $\mathbf{n}_t(1)$ of length J_1; $\mathbf{n}_t(2)$ of length J_2.

$J_h - 1$ separate equations, one for each possible time-since-last-adjustment. Specifically, each equation isolates the fraction of all plants that had productivity a_h, $h \in \{1, ..., M\}$, at the time of their last adjustment j periods in the past, do not adjust this period, and then draw random productivity a_m, $m \in \{1, ..., M\}$, at the start of the next period. This represents the number of plants that produce with employment $n_{jt}(h)$ in the current period and then enter the next period identified by $(n_{j+1,t+1}(l), a_m)$:

$$\theta_{j+1,t+1}(h,m) = \sum_{l=1}^{M} \theta_{jt}(h,l)[1-\alpha_{jt}(h,l)]\phi(l,m) \quad \text{for } j = 1, ..., J_h - 1. \tag{21}$$

Finally, there are M sets of equations describing future employments of those that had productivity a_h, $h \in \{1, ..., M\}$, at the time of their last adjustment:

$$n_{j+1,t+1}(h) = (1-d)n_{jt}(h) \quad \text{for } j = 0, ..., J_h - 1. \tag{22}$$

Equations (23) - (26) describe aggregate output gross of adjustment costs, aggregate labor demand and total adjustment costs in the economy. Aggregate output is total production across all establishments, which are grouped into two broad categories in equation (23). First, there are those plants with each current productivity $\{a_l\}_{l=1}^{M}$ that adjust to the target employment consistent with their productivity. Next, there are those with each current productivity a_l that do not adjust, but instead produce with the employment consistent with their productivity $\{a_h\}_{h=1}^{M}$ from the time they last did so.

$$\begin{aligned} Y_t &= \sum_{l=1}^{M} \Big[f\Big(n_{0t}(l), a_l, z_t\Big) \sum_{h=1}^{M} \sum_{j=1}^{J(h,l)} \theta_{jt}(h,l)\alpha_{jt}(h,l) \Big] \\ &\quad + \sum_{l=1}^{M} \sum_{h=1}^{M} \Big[\sum_{j=1}^{J(h,l)-1} f\Big(n_{jt}(h), a_l, z_t\Big) \theta_{jt}(h,l)[1-\alpha_{jt}(h,l)] \Big] \end{aligned} \tag{23}$$

Total employment demand is an analogous sum of the employments of adjusting and non-adjusting establishments:

$$\begin{aligned} N_t^D &= \sum_{l=1}^{M} \Big[n_{0t}(l) \sum_{h=1}^{M} \sum_{j=1}^{J(h,l)} \theta_{jt}(h,l)\alpha_{jt}(h,l) \Big] \\ &\quad + \sum_{l=1}^{M} \sum_{h=1}^{M} \Big[\sum_{j=1}^{J(h,l)-1} n_{jt}(h)\theta_{jt}(h,l)[1-\alpha_{jt}(h,l)] \Big]. \end{aligned} \tag{24}$$

Finally, economy-wide adjustment costs are the aggregate of those paid by establishments of each time-since-adjustment age that last adjusted to an employment consistent with productivity a_h and now have productivity a_l, again summing across past and current productivity levels,

$$Q_t = \sum_{l=1}^{M} \sum_{h=1}^{M} \sum_{j=1}^{J(h,l)} \theta_{jt}(h,l) \Gamma\Big(\alpha_{jt}(h,l)\Big), \tag{25}$$

where each $\Gamma(\alpha)$ in (25) represents the average adjustment cost paid per member in a given group, conditional on adjustment fraction α from that group,

$$\Gamma(\alpha) \equiv \int_0^{G^{-1}(\alpha)} x g(x). \tag{26}$$

6.2 Planning problem

The planning problem for the generalized partial adjustment model with persistent plant-specific productivities is straightforward given our aggregation above. Here, the aggregate state vector includes the $M^2 + M$ vectors that together describe the current distribution of establishments over employment and productivity, alongside current exogenous aggregate productivity, z: $S_t \equiv \Big[[\boldsymbol{\theta}_t(h,l)]_{h,l=1}^M, [\mathbf{n}_t(h)]_{h=1}^M, z_t\Big]$. The planner solves

$$W(S_t) = \max_{\mathbf{D}_t} \Big(u(c_t, 1 - N_t) + \beta E W(S_{t+1}|S_t) + \lambda_t [Y_t - Q_t - c_t] + w_t \lambda_t [N_t - N_t^D] \Big), \tag{27}$$

subject to (20) - (26), where

$$\mathbf{D}_t = \left\{ c_t, N_t,\ [n_{0t}(l)]_{l=1}^M,\ \Big[[\alpha_{jt}(h,l)]_{j=1}^{J(h,l)-1} \Big]_{h,l=1}^M,\ \Big[[\theta_{j+1,t+1}(h,l)]_{j=0}^{J(h,l)-1} \Big]_{h,l=1}^M \right\}.$$

The solution to this problem satisfies (20) - (26) and the constraints in (27) with equality, as well as a series of efficiency conditions that, after some algebra, may be written as follow. First, aggregate consumption and labor supply satisfy

$$\begin{aligned} \lambda_t &= D_1 u(c_t, 1 - N_t) \\ w_t \lambda_t &= D_2 u(c_t, 1 - N_t). \end{aligned}$$

Next, we describe the conditions determining target employments. For each plant-specific productivity level a_l, define $\omega_{0t}(l)$ to be the total establishments that currently have this productivity and adjust their employment;

$$\omega_{0t}(l) \equiv \sum_{h=1}^{M} \sum_{j=1}^{J(h,l)} \theta_{jt}(h,l) \alpha_{jt}(h,l), \ \text{ for } l = 1, ..., M.$$

The conditions identifying the optimal employment levels for each of these groups of adjusting establishments may then be written recursively as below in the $\sum_{h=1}^{M} J_h$ equations of (28) - (29). For $l = 1, ..., M$:

$$\left[D_1 f\Big(n_{0t}(l), a_l, z_t\Big) - w_t\right] + \beta(1-d)E\left(\frac{\lambda_{t+1}}{\lambda_t}\ \frac{\Omega_{1,t+1}(l)}{\omega_{0t}(l)}\ \mid\ S_t\right) = 0, \tag{28}$$

where, for each $h = 1, ..., M$,

$$\Omega_{jt}(h) \equiv \sum_{l=1}^{M} \theta_{jt}(h,l)[1-\alpha_{jt}(h,l)]\left[D_1 f\Big((1-d)^j n_{0t}(h), a_l, z_t\Big) - w_t\right] \tag{29}$$

$$+\beta(1-d)E\left(\frac{\lambda_{t+1}}{\lambda_t}\Omega_{j+1,t+1}(h)\ \mid\ S_t\right),\ \text{ for } j = 1, ..., J_h - 1.$$

These conditions closely parallel those in the model without plant-level productivity shocks. As there, the marginal effects of the current employment choice on future production and wage payments continue for as long as a plant does not re-adjust. The second term in (28) reflects the probability-weighted sum of future effects for any member of the $\omega_{0t}(l)$ group of adjusting plants. Any such plant may enter date $t+1$ with productivity a_1 and produce without readjusting its employment with probability $\frac{\theta_{1,t+1}(l,1)}{\omega_{0t}(l)}[1-\alpha_{1,t+1}(l,1)]$; with probability $\frac{\theta_{1,t+1}(l,2)}{\omega_{0t}(l)}[1-\alpha_{1,t+1}(l,2)]$, it will have productivity a_2 and not adjust, and so forth. The collections of equations in (29) summarize the resulting marginal effects from $t+1$ and forward until date $t+J_l$, the date by which the currently adjusting plant will re-adjust its employment with certainty if it has not already done so.

The conditions (30) determining optimal adjustment fractions from within each group of plants are also analogous to those in section 4. Within each group that had productivity a_h at the time of last adjustment, $h \in \{1, ..., M\}$, and now have productivity a_l, $l \in \{1, ..., M\}$, the fraction adjusting from each time-since-last-adjustment subgroup will satisfy

$$\xi\Big(\alpha_{jt}(h,l)\Big) = v_{0t}(l) - v_{jt}(h,l) \text{ for } j = 1, ..., J(h,l) - 1, \tag{30}$$

where $\xi(\alpha) \equiv G^{-1}(\alpha)$. Each adjustment fraction equates the marginal cost paid to adjust the last plant from a given group to the net value of moving that plant into the adjustment group associated with its current productivity, given the production-time values of each plant type expressed recursively below.

The value of any adjusting plant with current productivity a_l is, for each $l = 1, ..., M$:

$$\begin{aligned} v_{0t}(l) &= \Big[f\Big(n_{0t}(l), a_l, z_t\Big) - w_t n_{0t}(l)\Big] + \beta E\Bigg[\frac{\lambda_{t+1}}{\lambda_t}\sum_{m=1}^{M}\phi(l,m)\Big(\alpha_{1,t+1}(l,m)v_{0,t+1}(m) \\ &\qquad + [1-\alpha_{1,t+1}(l,m)]v_{1,t+1}(l,m) - \Gamma\Big(\alpha_{1,t+1}(l,m)\Big)\Big) \mid S_t\Bigg]. \end{aligned}$$

This includes the plant's current profit associated with $(n_{0t}(l), a_l)$ at production time and its discounted probability-weighted continuation value. (For example, at date $t+1$, the plant will draw productivity a_2 with probability $\phi(l, 2)$. In that case, its expected fixed cost payment will be $\Gamma\Big(\alpha_{1,t+1}(l,2)\Big)$, and it will produce with either the target employment $n_{0,t+1}(2)$ with probability $\alpha_{1,t+1}(l,2)$ or $n_{1,t+1}(l)$ with probability $1-\alpha_{1,t+1}(l,2)$.) Next are the values associated with units that do not adjust. For each (h,l) productivity pair, and for each $j = 1, ..., J(h,l)-1$, the value of a nonadjusting plant identified by $(n_{jt}(h), a_l)$ at production time in date t is

$$\begin{aligned} v_{jt}(h,l) &= \Big[f\Big(n_{jt}(h), a_l, z_t\Big) - w_t n_{jt}(h)\Big] + \beta E\Bigg[\frac{\lambda_{t+1}}{\lambda_t}\sum_{m=1}^{M}\phi(l,m)\Big(\alpha_{j+1,t+1}(h,m)v_{0,t+1}(m) \\ &\qquad + [1-\alpha_{j+1,t+1}(h,m)]v_{j+1,t+1}(h,m) - \Gamma\Big(\alpha_{j+1,t+1}(h,m)\Big)\Big) \mid S_t\Bigg]. \end{aligned}$$

Given the current aggregate state, S_t, and the values of $J(h,l)$, the evolution of this economy is fully described by a system of $\Big[7 + 2\sum_{h=1}^{M}\sum_{l=1}^{M} J(h,l) + (M+2)\sum_{h=1}^{M} J_h - M^2\Big]$ first-order stochastic difference equations. As a result, the economy's aggregate dynamics may be solved as a local approximation around the steady state using standard linear systems methods. Thus, while extending our framework to allow for additional heterogeneity such as the plant productivity shocks considered here certainly increases the number of equations involved, it requires no more complicated solution algorithm than that of the basic framework lacking these richer elements.

7 Concluding remarks

In the preceding sections, we have developed a new partial adjustment model for labor demand that can be employed without apology for its microeconomic implications and is tractable for dynamic general equilibrium analysis. Our generalized partial adjustment model is consistent with 5 stylized facts: (1) employment adjustment at the establishment is discrete and occasional, (2) aggregate employment is smooth and gradual, (3) individual plants' probabilities of adjustment vary over time in response to aggregate conditions, (4) these adjustment probabilities are functions of the difference between plants' actual and target employment and (5) movements in aggregate employment are largely driven by movements in aggregate factors.

The last stylized fact has led us to focus our quantitative examples on situations where idiosyncratic uncertainty at the plant level is transitory, and there are no additional sources of heterogeneity. Existing empirical research suggests that such factors are of secondary importance in explaining movements in aggregate employment. A benefit to our abstraction is that we are able to develop a generalized (S, s) model of establishment-level labor adjustment that rationalizes existing empirical work which has heretofore assumed state-dependent adjustment hazards. Moreover, we have shown that our method allows convenient aggregation of the discrete adjustment actions of a heterogeneous distribution of production units into a planning problem.

Using our generalized partial adjustment model, we have analyzed the dynamics of employment under two alternative assumptions about the wage rate and interest rate. We began by assuming that both prices were fixed, while productivity fluctuated exogenously. Next, we considered a simple general equilibrium formulation in which these prices were endogenously determined and hence varied with changes in productivity. The dynamics under these two formulations are quite distinct, and they lead to somewhat different conclusions about how well the aggregate dynamics of our model are approximated by the traditional partial adjustment model. Previous research in this area has been conducted almost exclusively under the assumption of exogenous prices, given the complications presented by nontrivial heterogeneity in production. An important contribution of our approach lies in its ability to limit such complications, thereby facilitating general equilibrium analysis.

Finally, while we have chosen to abstract from additional sources of plant-level heterogeneity in the numerical examples here, pending a detailed calibration exercise, we have shown how the addition of persistent plant-specific shocks is a straightforward extension of our current framework. Thus, we have provided a tractable basis for future research into the dynamics of factor adjustment and, indeed, other questions where discrete individual decisions interact with persistent stochastic sources of heterogeneity.

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Figure 1a: Steady State Adjustment Fractions

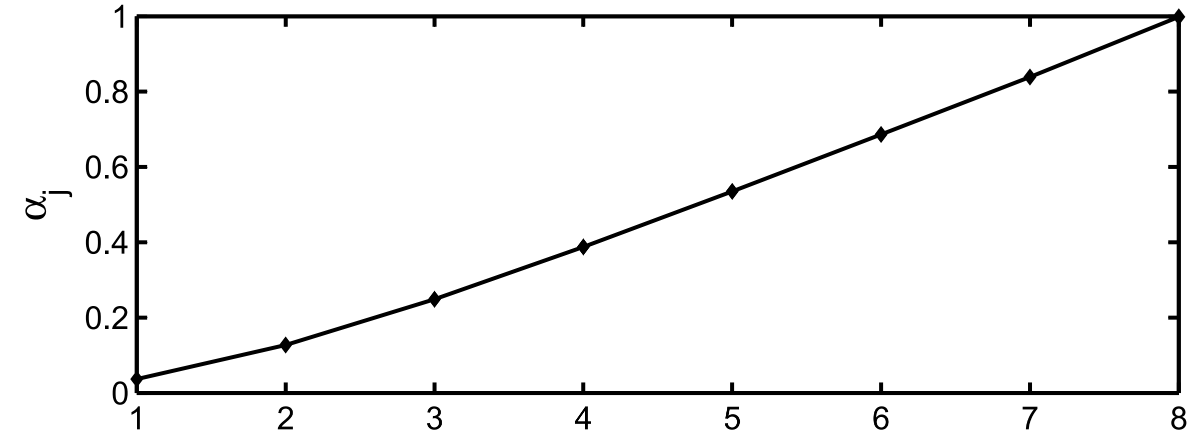


Figure 1b: Steady State Distribution

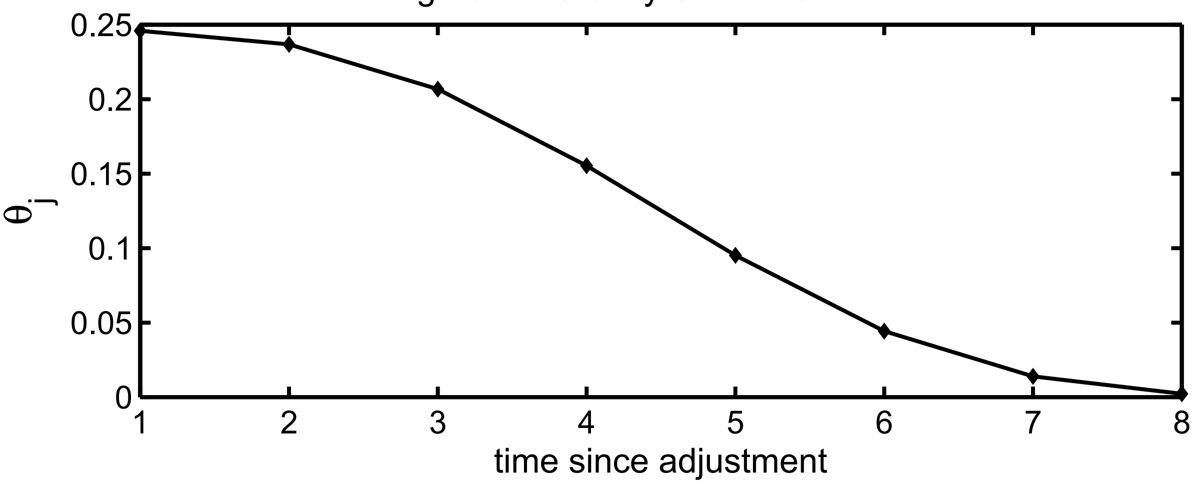


Figure 2a: Market Employment

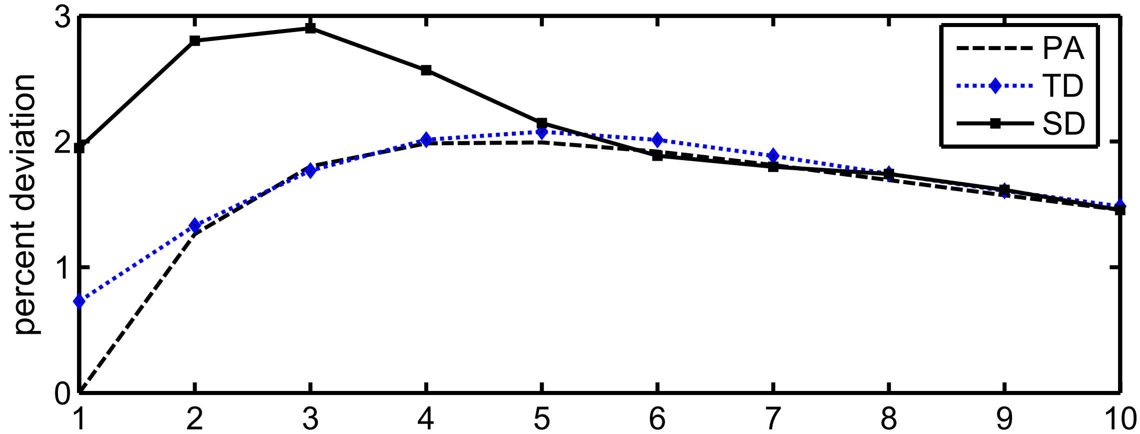


Figure 2b: Market Output

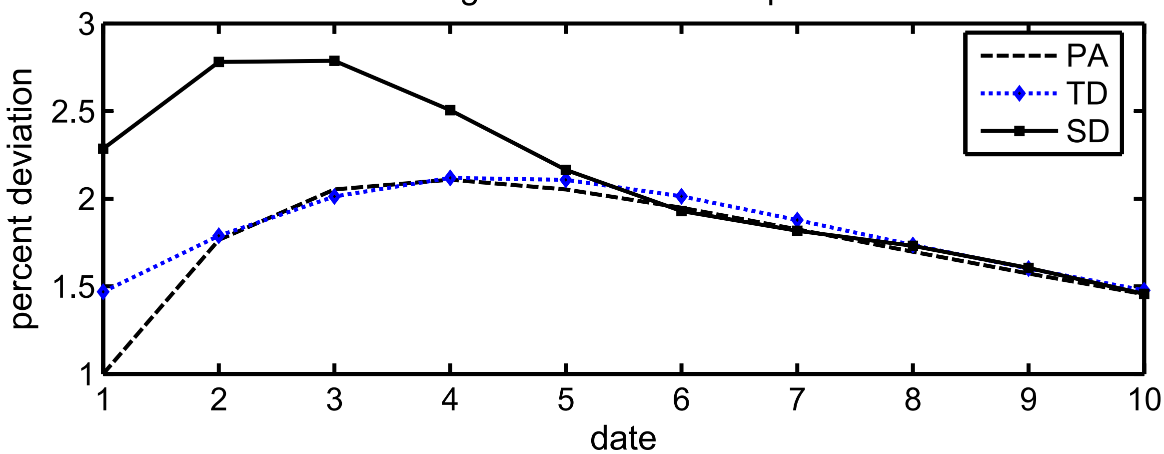


Figure 3a: Total Adjustors

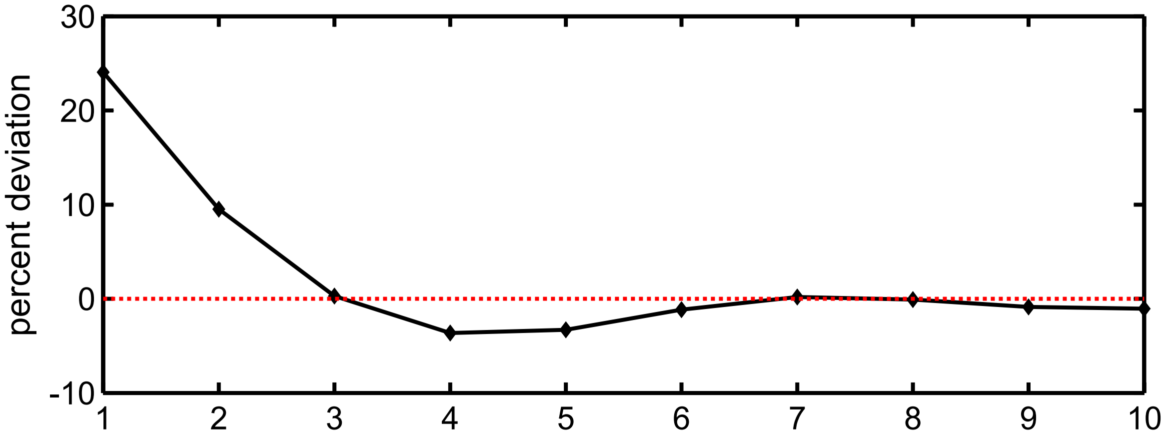
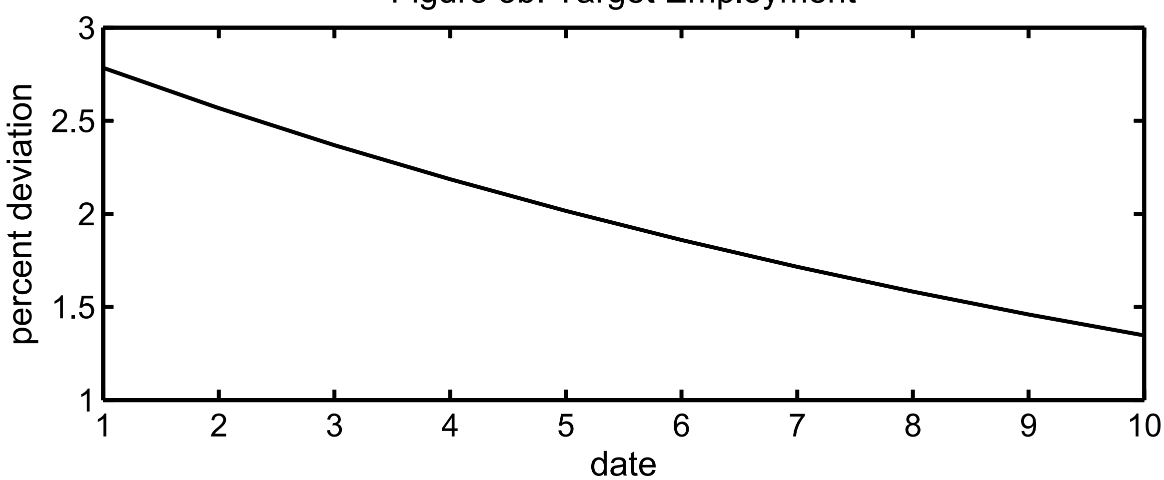


Figure 3b: Target Employment



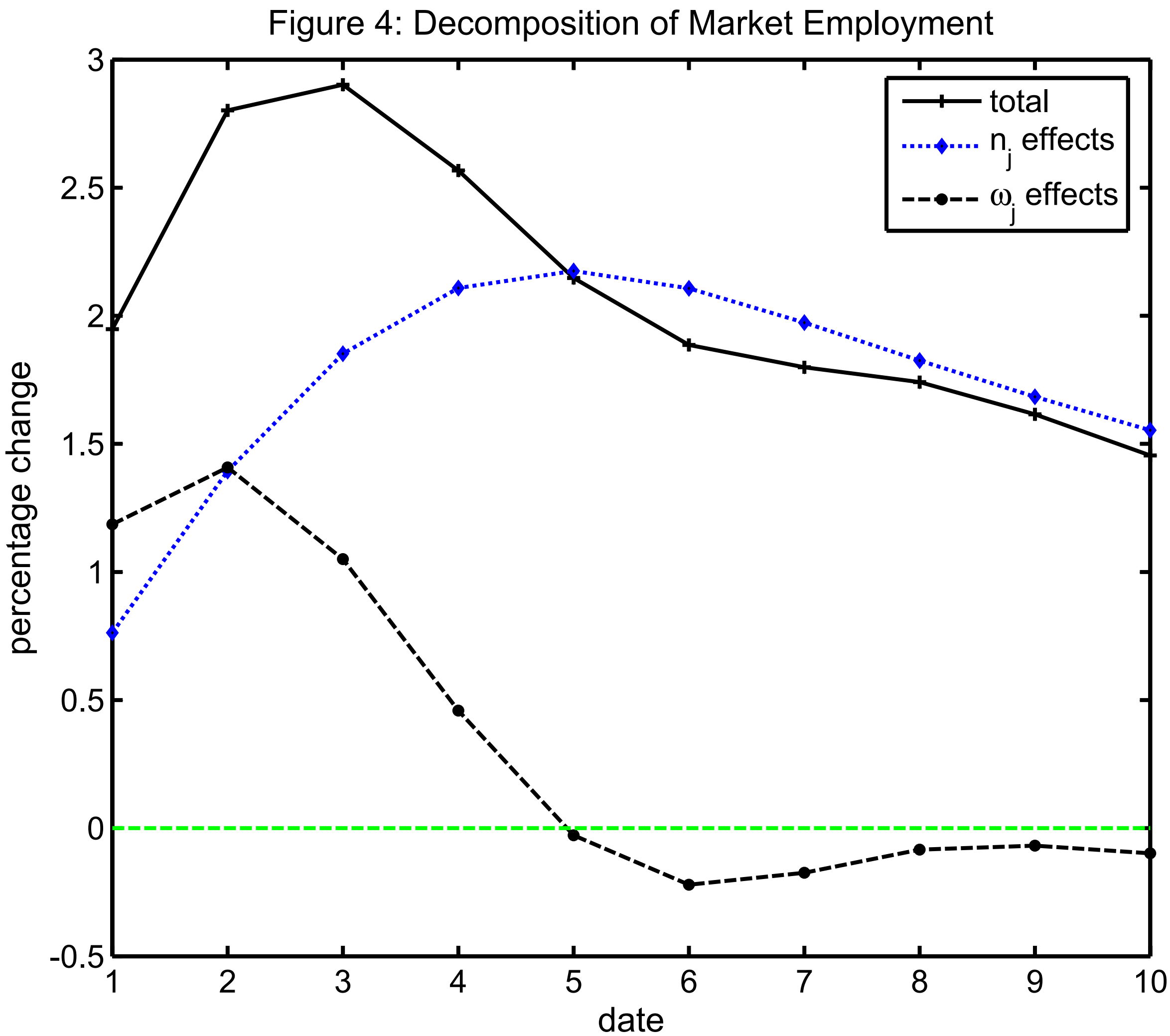


Figure 4: Decomposition of Market Employment
total
n_j effects
ω_j effects
percentage change
date
3
2.5
2
1.5
1
0.5
0
-0.5
1
2
3
4
5
6
7
8
9
10

Figure 5: Effects of Equilibrium

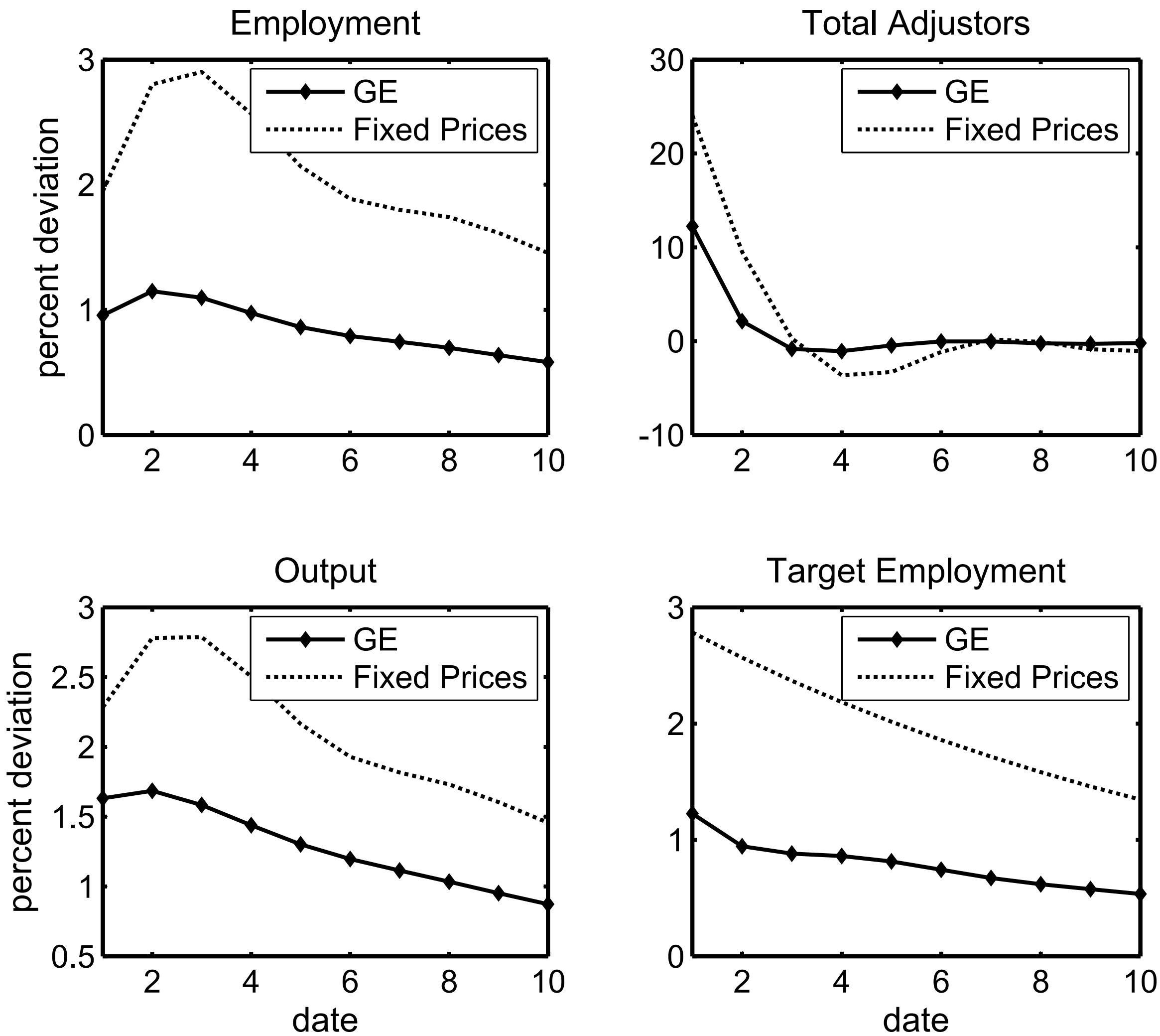


Figure 6a: Employment in General Equilibrium

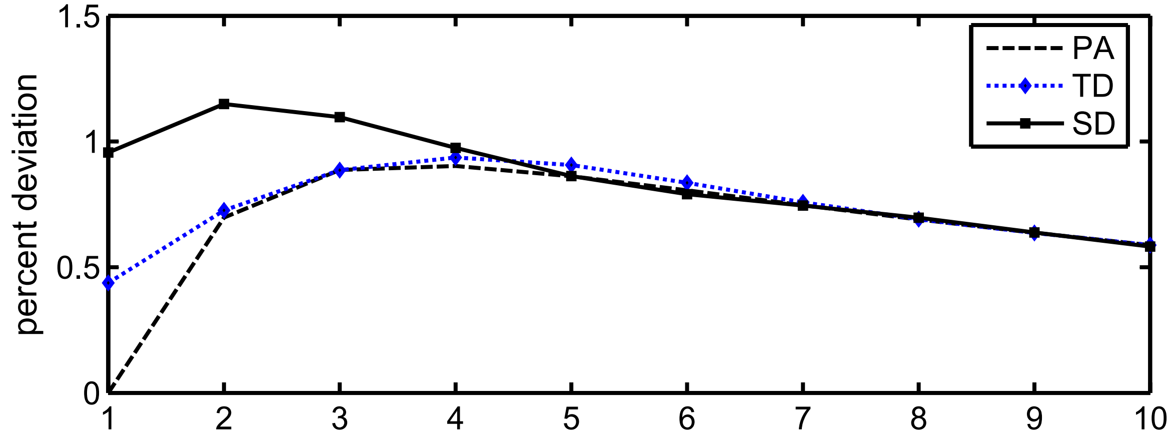


Figure 6b: Output in General Equilibrium