Do Sunspots Produce Business Cycles?

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Abstract

I compare the canonical sunspots and real business cycle models of aggregate fluctuations. The sunspots model, distinguished by production externalities, is better able to reproduce the typical spectral shape of growth rates found in the data. However, it generates excessive investment volatility and overstates high frequency behavior in employment, investment and output series. The introduction of adjustment costs, in conjunction with separate externality parameters to capital and labor inputs, can reduce these weaknesses substantially, though this may require the assumption of an implausible level of increasing returns.

1 Introduction

Since the early 1980's, the real business cycle model (RBC) has been the dominant organizing paradigm for quantitative equilibrium business cycle analysis.[1] The framework has been relatively successful in explaining the comovements of real variables, the relative volatilities of output, investment and consumption, and the procyclical labor productivity observed in the data. Nonetheless, it has suffered several

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[1] Variants of the basic real business cycle model are described in Hansen (1985), Prescott (1986), and King, Plosser and Rebelo (1988).

criticisms. First, this basic neoclassical model overpredicts the actual correlation between productivity and output, and it dramatically overstates the correlation between labor input and wages (Baxter and King (1991); Christiano and Eichenbaum (1992)). Next, as it provides limited propagation to the cycle, the model economy's behavior roughly approximates that of the exogenous stochastic technology process; hence, a highly persistent shock is required to generate the persistence of the cycle.[2] Another difficulty with the standard paradigm is the problem of finding supporting evidence for the technology shocks that drive the RBC model economy.[3] As a result of such criticisms, business cycle researchers have been led to consider an alternative class of model built on indeterminacy of competitive equilibria.

Several researchers have shown that modifications to the standard neoclassical growth model to include production externalities or imperfect competition can deliver sufficient aggregate increasing returns to yield indeterminacy of the dynamical system. As a result, these *animal spirits* or *sunspots* models are able to generate business cycle fluctuations in the absence of any shocks to fundamentals; instead cycles are the result of iid expectational shocks.[4] In contrast to the standard neoclassical model, this framework allows for a pair of complex, stable eigenvalues yielding dampened oscillations. In other respects, the sunspots model is similar to the real business cycle economy.

I contrast the animal spirits model of Farmer and Guo (1994), characterized by externalities to aggregate inputs, with the indivisible labor business cycle model of

[2] See Christiano (1988), Rouwenhorst (1991), Eichenbaum (1993), and Cogley and Nason (1995).

[3] Gordon (1993) argues that all correlation between productivity and output is due to measurement error in capital and hours, and that there is no actual contribution of technology shocks to output fluctuation. Additionally, Cochrane (1994), using Beveridge-Nelson detrending, finds less than a .002 percent contribution of technology shocks to detrended output variance.

[4] Benhabib and Farmer (1994) and Farmer and Guo (1994) provide one-sector models of sunspots in the presence of input externalities in final goods production or, equivalently, monopolistic competition and increasing returns in intermediate inputs production. Other examples of the framework include the monopolistic competition environments of Gali (1994) and Beaudry and Devereux (1994), multi-sector input externalities models of Benhabib and Farmer (1996) and Perli (1994), and human capital externalities models, as in Xie (1994) and Benhabib and Perli (1994).

Hansen (1985). The Farmer and Guo and Hansen models are representative of their respective classes, sunspots versus neoclassical real business cycle. These reference models have the advantage of being equivalent in every aspect except for the level of returns in production. I choose these, over less comparable variants, to represent the canonical real business cycle model and canonical sunspots model. In the concluding remarks, I discuss the generality of the results reported here, in particular the extent to which they are likely to apply to richer versions of the sunspots model.

After providing a brief summary of the Farmer and Guo and Hansen models in section 2, I isolate the propagation mechanism in each model through a comparison of impulse responses in section 3. Serious consideration of the sunspots economy as an alternative model of the business cycle requires a careful assessment of its ability to generate realistic time series relative to the existing paradigm. Section 4 briefly reviews the Watson (1993) measures of fit technique, and describes how it may be used to generate a quantitative evaluation of the performance of both models relative to the data. Extending the original comparison of Farmer and Guo (1994), section 5 presents a comparison of spectra generated by the sunspots and RBC models, along with the results of Watson's test. The final section provides concluding remarks.

I find that the sunspots model is more successful in producing the typical spectral shape of growth rates seen in the data; its output, employment and investment growth series display a hump inside the business cycle band, while the benchmark RBC model has no such feature and concentrates more of its power in the low frequencies. However, the sunspots economy exhibits excessive investment volatility at all but the lowest frequencies and generates too much high frequency activity in output, employment and investment.[5] These results raise the question, "are excessive investment volatility and high frequency action necessary to the sunspots framework's success in generating a spectral hump?" To explore this, I modify the baseline sunspots model to allow for separate externality parameters to capital and labor inputs. Varying externalities over a wide range of values, I find that these two

[5] Both features also occur in the RBC economy, but they are minor relative to the sunspots counterparts.

problems are persistent flaws of the model. However, a further modification allowing for capital adjustment costs suggests that these features may be reduced substantially when adjustment costs are small and the externality to labor is large.

2 Reference Models

This section reviews the sunspots model of Farmer and Guo (1994) and the Hansen (1985) real business cycle model, which serves as our benchmark.

The aggregate technology of the sunspots model exhibits increasing returns to scale, while the RBC aggregate production function has constant returns. However, within both models, firm-specific production is characterized by constant or diminishing returns, and firms behave competitively and have well defined maximization problems. Output of the single consumption/investment good, Y_t, is given by (1), where z_t is current total factor productivity, K_t is firm-specific capital, N_t is labor, and economy-wide averages of capital and labor are given by $\overline{K}_t$ and $\overline{N}_t$.

$$Y_t = z_t K_t^a N_t^b \left(\overline{K}_t^{a\Psi} \overline{N}_t^{b\Psi} \right) \tag{1}$$

Each individual firm takes $\overline{K}_t$ and $\overline{N}_t$ as given. Capital and labor shares of output are, respectively, a and b, where $a > 0$, $b > 0$, and $a + b \leq 1$.[6] The parameter Ψ captures the external effects of inputs in the sunspots model. When $\Psi = 0$ and $a + b = 1$, we have the constant returns production technology of the RBC model.

Defining time t consumption as C_t and the constant discount factor as ρ, the representative agent's expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \rho^t \left(\log C_t - \frac{A N_t^{1-\gamma}}{1-\gamma} \right), \tag{2}$$

where $\gamma \leq 0$. As seen in the calibration table below, both reference models share the indivisible labor specification $\gamma = 0$ of Hansen (1985). Denoting investment at time

[6] When $a+b < 1$, there are nonzero profits remunerated to the representative agent in each period.

t by I_t, each economy is subject to a sequence of goods constraints,

$$Y_t \geq C_t + I_t \,; \quad t = 0, 1, \ldots \,, \tag{3}$$

capital accumulation constraints,

$$(1 - \delta)K_t + I_t \geq K_{t+1}; \quad t = 0, 1, \ldots \,, \tag{4}$$

and non-negativity constraints on consumption, investment and labor supply.

The behavior of each economy is described by the standard marginal conditions resulting from the maximization of (2) subject to (1), (3), (4) and the initial capital stock K_0, along with the aggregate constraints. In equilibrium, $K_t = \overline{K}_t$ and $N_t = \overline{N}_t$, so that, defining $\alpha = (1 + \Psi)a$ and $\beta = (1 + \Psi)b$, aggregate production is $Y_t = z_t K_t^{\alpha} N_t^{\beta}$, and the aggregate goods and capital accumulation constraints are given by (3) and (4). I log-linearize the aggregate efficiency conditions and constraints in order to examine the dynamics of the models. The solution method differs according to each model's stability properties. Determinacy of the competitive equilibrium, given the presence of one state variable K and one co-state λ (the shadow value of output), requires a pair of initial conditions. The RBC economy exhibits saddle-path stability; it is characterized by one stable, and one unstable, eigenvalue. Thus, the standard solution for linear rational expectations systems applies; the unstable eigenvalue is solved forward to derive an initial condition for the co-state which, coupled with the initial condition for capital, is sufficient to yield determinacy.[7] The sunspots economy, by contrast, is characterized by a degree of increasing returns consistent with a stable steady state.[8] The associated absence of an unstable eigenvalue means that the economy lacks one initial condition, and competitive equilibria are indeterminate. Exploiting this indeterminacy, Farmer and Guo (1994) allow endogenous

[7] See Blanchard and Kahn (1980) or King, Plosser and Rebelo (1988) for further discussion of this solution method.

[8] Stability requires that the labor demand curve be upward sloping and steeper than the labor supply curve, or that $\beta - 1 > |\gamma|$. This necessary condition is achieved through a combination of sufficient increasing returns, a large share to labor in production, and a highly elastic labor supply. See Aiyagari (1995) for further discussion.

fluctuations in the co-state; in practice, these fluctuations are driven by mean zero shocks, consistent with rational expectations, to the agent's linearized Euler equation.

While the basic structure of these two models is identical, I have noted that they differ in their degree of aggregate returns. The frameworks are further distinguished by their driving processes. The sunspots model is driven by self-fulfilling shocks to income beliefs, or "animal spirits," modelled as independent and identically distributed, mean zero, variance σ^2 innovations to λ, with the variance of the mean zero innovation to the technology shock process held at zero. For the RBC model, the technology shock is a first order autoregressive process $\widehat{z}_t = \theta \widehat{z}_{t-1} + \varsigma_t$. The innovation term driving this process is $\varsigma_t \sim N(0, \sigma_\varsigma^2)$, and the variance of the sunspot shock is zero. The parameter values below are taken from Farmer and Guo (1994).[9]

Parameter	γ	a	b	Ψ	δ	ρ	σ	θ	σ_ς
Sunspots Model	0	.23	.70	.724	.025	.99	.00217	n/a	0
Technology Shock Model	0	.36	.64	0	.025	.99	0	.95	.007

Note that the calibration chosen for the animal spirits model assumes a degree of external increasing returns for the production technology sufficient to generate two complex, stable eigenvalues for the dynamical system. The resulting pair, $.92 \pm .1i$, implies cyclical fluctuations of period 58.59 quarters per cycle. In contrast, the calibration chosen for the RBC economy assumes constant returns to scale and no externalities, so it exhibits saddle-path stability.

[9] Farmer and Guo use a model of monopolistic competition that is equivalent to the model of externalitites in production presented here. Their model sets a value for the monopoly parameter at $\chi = 0.58$. The equivalent parameter selection here is $\Psi = \chi^{-1} - 1$.

3 Impulse Responses

Figures 1 and 2 present impulse responses for the animal spirits and RBC economies, generated using the dynamic multipliers implied by the linear system of each model.[10] Figure 1 depicts the sunspots model for the first 50 quarters following a one standard deviation animal spirits shock, with the technology shock held at its mean of zero. Similarly, figure 2 shows 50 quarters of results for the RBC model in response to an innovation of one standard deviation to the technology shock in the absence of any animal spirits shock. Notice that, just as the real business cycle model essentially mimics the exogenous shock process in its responses, the sunspots responses closely resemble the implied path of total factor productivity for the model. This is in keeping with Kamihigashi's (1995) finding that a sunspots model is observationally equivalent to an RBC economy in which the productivity shocks take on the values of the function capturing external economies, and it supports his conclusion that the success of a sunspots economy in generating realistic impulse responses implies the same success for real business cycle models employing richer stochastic structures.

While capital accumulation propagates shocks in both models, the mechanics of the animal spirits cycle are quite distinct from those of the real business cycle model. The shock process driving cycles in the sunspots economy lacks persistence; capital is the sole propagating mechanism for the animal spirits cycle. Thus, across series, we see that the RBC responses are slow and persistent when viewed alongside the sunspots responses. In contrast to the RBC model, the sunspots model produces mild oscillatory dynamics, a result of the combination of a high labor supply elasticity, a high labor share of output, and large increasing returns to the labor input, which, taken together, yield a pair of stable complex eigenvalues in the dynamic system. These dampened oscillations imply pseudo-cyclical behavior that generate a concentration of spectral power within the business cycle band, so that the sunspots model achieves a closer relation to the data's humped shape when we consider the growth

[10] Such figures are also available in Farmer and Guo [14]; however, the impulse responses there are generated using multipliers derived from a VAR estimation of each simulated economy. Thus, the oscillatory features discussed there could be magnified or mitigated by the VAR estimates.

spectra below.[11]

Though the real business cycle economy moves more slowly than the sunspots economy, their impulse responses are qualitatively similar. Both models' initial responses can be broken into three consecutive stages: (i) the impact date in which consumption, employment, output, investment and the real wage rise, (ii) the periods during which consumption and wage continue upward while other variables fall towards trend,[12] and (iii) those quarters in which all variables are declining. I explore each of these stages below.

The impulse for the sunspots economy is a spontaneous change in agents' expectations. The representative agent expects higher permanent income; therefore current consumption and leisure demand rise. This forces the labor supply curve upward, which, under ordinary circumstances, would yield reduced employment. However, strong increasing returns in production implies an upward sloping marginal product of labor schedule, so that both wages and employment rise in equilibrium. This boosts output, fulfilling the agent's expectations. Consumption rises, and, due to permanent income effects, there is an investment boom. This sequence of impact events is quite different from the origins of the cycle within a real business cycle economy, where a technology shock shifts the marginal product of labor schedule outward, causing higher equilibrium employment, output, consumption and investment.

Beyond impact, consumption and wage series continue to rise, tracing the first half of a hump, while other variables return monotonically towards their steady state levels. In both models, this is generated by the substantially increased capital stock resulting from the earlier investment choices. This high capital stock raises the marginal product of labor, shifting the labor demand curve upward. However, it also lowers the marginal product of capital, reducing the relative price of current goods.

[11] It is possible for the model to generate deterministic cycles for some combinations of capital and labor externality parameters. For example, there are complex eigenvalues of modulus 1 for capital externality and labor externality choices in the region of .11 and 1.05. While this is a large labor externality, total returns are 1.69, which is quite close to the benchmark case.

[12] Note that, for each model, the specification of preferences as linear in leisure leads to proportional consumption and wage series.

The falling interest rate encourages agents to curb their investment relative to impact, and raises demand for consumption and leisure further. This substitution effect dominates the negative income effect of the anticipated decline in output upon leisure demand, so that the labor supply curve shifts upward. Within the sunspots economy, the negative employment effect of the upward (inward) labor demand shift outweighs the positive effect of the labor supply shift, so that equilibrium labor input begins to decline, while consumption and wages continue to rise. Further, because labor's importance in production is large in this model, the effect on output is quite negative, even while the capital input is high. Conversely, an increased wage and reduced employment result in the RBC economy because the labor supply effect outstrips the upward (outward) shift of the labor demand curve.

Eventually, the reduced investment of the second stage causes the capital stock to peak. At this time, the labor demand curves of the two economies begin shifting downward. This tends to reduce employment in the RBC model, but raise it in the sunspots model. Simultaneously, the falling capital stock, which will further reduce output, increases the real interest rate. The resulting income and substitution effects tend to reduce consumption and leisure, shifting the labor supply curve downward. Within the RBC framework, it is the effect of the falling labor demand curve that dominates, while the labor supply shift dominates the animal spirits response. Across both models, consumption, wages and capital begin to fall, and employment, output and investment continue to decline. Finally, the real business cycle model exhibits some overshooting in employment and investment, as agents save less and enjoy more leisure in order to deplete excess capital stock, while the sunspots economy returns to steady state through a series of dampened oscillations.

4 A Measure of Fit

One important motivation for the interest in increasing returns/animal spirits models as an alternative to constant returns/technology shock models has been a desire to provide a better match with the data. Farmer and Guo write:

> We show that our economy can explain the contemporaneous correlations of output, employment, investment, and consumption in U.S. time series with about the same degree of precision as the standard real business cycle model and that it is more successful at capturing the *dynamics* of U.S. data.[13]

This may be interpreted as an assertion that the autocovariances of the animal spirits model are closer to autocovariances generated by the data than are those of the technology shock model. As any statement about the model's autocovariances in the time domain may be translated into an equivalent statement in the frequency domain, I consider a spectral decomposition of the aggregate time series implied by the sunspots model in an effort to evaluate this claim, comparing these to post-war U.S. data spectra as well as spectra generated by the real business cycle benchmark.

In the absence of a quantitative criterion, the task of evaluating the success of the sunspots model relative to the technology-shock model would rely on such a comparison of the spectral decompositions. While this is certainly the starting point for a comparative exercise, it is qualitative in nature. To provide a more precise measure of the relative success of the sunspots model, I turn to Watson's technique of deriving the minimum variance approximation error associated with the models.[14]

Watson provides a method for quantitatively evaluating the overall fit of calibrated models and applies it to the business cycle model of King, Plosser, and Rebelo (1988). Using this approach, I test the goodness of fit of the animal spirits economy, and contrast the results with those of the benchmark real business cycle model. This method complements the more subjective appraisals upon which researchers have previously relied.

Watson's technique overcomes the difficulty of the typical equilibrium business cycle model's lack of a complete underlying probability structure necessary to interpret differences between model and data using standard statistical methods, and allows

[13] Farmer and Guo (1994), page 43.

[14] The appendix describes the simplest application of Watson's technique and discusses how it is used to evaluate the goodness of fit for the sunspots and real business cycle models.

both inference and a measure of the fit for such models. His approach is to append the model with the minimum amount of approximation error required to match the second moment properties of the data. Minimum approximation error may be interpreted as a measure of the degree of model abstraction. If the variance of this error is large, we can infer that the model does a poor job of explaining the observed data. If it is small, the model can fit well under some restrictions on the joint autocovariance of the model and data.

Application of the method involves minimizing the trace of the spectral density matrix for the abstraction error, frequency by frequency, by choice of the cross spectrum between data and model variables. The ratio of the variance of the required error to that of the actual time series provides a relative mean square approximation error measure, or $RMSAE$, at each frequency, from which we can compare models not simply on overall performance, but also within the business cycle band, frequencies of 6-32 quarters per cycle. In addition, for each economic time series, the relative mean square approximation errors can be integrated over frequencies to form a lower bound on the model's average relative mean square approximation error, analogous to finding an upper bound on the R^2 in a regression of the data on the model.

5 Results

I first evaluate the relative abilities of the sunspots and RBC models to fit aggregate time series on consumption, employment, output, productivity, and investment using a qualitative comparison of the model and data growth spectra in figure 3.[15] (Figures 4-5 display spectra for Hodrick-Prescott filtered and exact bandpass filtered consumption, output and investment series. I choose to focus on growth rate spectra here for sake of brevity.) The estimated data spectra display characteristics similar

[15] Data spectra are derived from the series used in King, Plosser, Stock and Watson [20]. The data include private GNP, real consumption spending, real fixed investment, private nonagricultural labor hours, and the real wage. The estimates are obtained via a vector autoregression, with an error correction matrix to account for cointegration between output, consumption and investment series.

to the “typical spectral shape for growth rates” discussed by King and Watson.[16] The power spectra of all variables, excepting the wage, have roughly the same shape, each exhibiting a hump. All series have relatively high power in the business cycle frequencies, those between .03 and .167 cycles per quarter, and relatively less power in the lower band. The business cycle frequencies account for between 50 and 70 percent of total variability for labor, investment and output, while they account for only about one-third in the consumption and wage series. All variables have spectral peaks within the business cycle range, each peaking around .05, which corresponds to cycles of 5 years length. While the wage series is roughly comparable to the other four series in the low frequency range, it displays relatively less business cycle power and more high frequency variation. Finally, the spectral estimates for output, investment and consumption display the typical relative magnitudes: the average height of the investment spectrum within the business cycle frequencies is significantly greater than that of output, which exceeds that of consumption. In particular, the height of the investment spectrum is about seven times that of output, while output is about 5 times that of consumption.

Examining the growth rate spectra implied by the real business cycle and sunspots models, we see that neither model mimics the data spectra in a satisfactory manner. For the output, employment and investment series, both models contain about half the necessary concentration of power in the business cycle band (28% in RBC and 29% for sunspots) and too much power in the high frequencies, as both models lack the data's sharp declines through the business cycle band. Further, within the business cycle band, we see that the ratios of power between output and consumption and between investment and output are exaggerated relative to the data, evidence of overstated consumption smoothing behavior within both frameworks. A notable difference between the two models' spectral shapes is that the sunspots economy exhibits a small hump within the lower bound of the business cycle band, absent in the RBC economy.

Recall that, given the assumption of indivisible labor, the series for consumption

[16] See King and Watson [21] page 7.

and the real wage are proportional. For both models, their spectral shapes are distinct from those of the other series, exhibiting pronounced humps in the low frequencies. Through the business cycle band, each generally appears as a phase-shifted version of the data, though the RBC model's persistent shock process causes its spectra to shift more power into lower frequencies than the sunspots spectra. Beyond the low frequencies, each generates too little power to match the data; virtually all power there is attributable to the first difference filter.

We may interpret the differences across model spectra as follows. Recall that the animal spirits model is driven by an iid shock process, while the RBC process is highly persistent. All else being equal, this would give the real business cycle model much greater power in the low frequencies. The sunspots model adds persistence to the economy, and hence low frequency power to the spectra, using increasing returns in production to magnify the investment propagation mechanism. The degree of increasing returns, however, is not sufficient to fully offset the lack of a persistent shock process. Consequently, the sunspots model has a lower concentration of power in the low frequencies than does the real business cycle economy, and a slightly higher concentration within the business cycle band, which is more consistent with the data spectra. However, it is this very feature which generates the model's worst spectral failing. Because its shocks are iid, capital provides the only dynamic link; the model must rely much more heavily on investment for its propagation than does the technology shock framework. The result is an amplified investment spectrum that dwarfs the data everywhere, remaining essentially flat from the business cycle band upward, so that the gap between model and data is ever widening at increasing frequencies. Finally, the animal spirits model's most apparent success is the hump present in its employment, output and investment spectra, a result of the oscillatory behavior associated with the presence of complex eigenvalues in the model's dynamic system. However, because the complex components of the eigenvalues are small, this hump is not very pronounced.[17]

[17] Because employment and investment exhibit some overshooting for the real business cycle economy impulse responses, the level spectra for these series also exhibit a hump. However, its placement

Turning to a more careful look at the spectral results, we examine relative mean square approximation errors for each model in figure 6. Because RMSAEs are invariant to linear filtering at each frequency, the choice of filter is unimportant. Each choice of primary variable in error minimization presents a new set of RMSAEs, but there are certain broad conclusions to be drawn from any of these studies. Thus, I display only one set of results, that in which output is chosen for error minimization. Within each separate study, the shapes of the RMSAEs are virtually identical across models, reflecting close spectral shapes.

Recall that the RMSAE can be interpreted as yielding the upper bound on an R^2 from a regression of a data series on its model counterpart at each particular frequency. In this sense, the sunspots investment series can be said to fail completely in all but the very lowest frequencies, even in the case where investment is chosen as the variable of interest,[18] since its RMSAEs greatly exceed 1. This feature reflects the extreme investment volatility of the animal spirits economy, also present in a less dramatic form for the RBC figures. Similarly, because both models' employment and output spectra retain too much high frequency power, failing to match the data's decline, these RMSAEs generally exceed 1 everywhere above the business cycle band.[19] As the sunspots employment spectrum has more high frequency power than does the benchmark model, its high frequency RMSAEs exceed the RBC counterparts by a small margin in all cases. Finally, for both frameworks, note that consumption RMSAEs usually remain within acceptable levels in the business cycle band. These successes correspond to frequencies at which model and data spectra are rapidly declining and then fairly flat. RMSAEs for the wage series are typically above or close to 1; these reflect the general inability of both frameworks to capture the data's spectral shape, and their insufficient power above the low frequencies. Both consumption and wage RMSAEs spike within the very low frequencies, because the

in very low frequencies causes it to be eliminated by the first difference filter.

[18] The scale of any particular variable's RMSAE is reduced substantially when it is chosen as the primary variable of interest.

[19] For each of these series, the one exception is the case in which that variable has been chosen for error minimization.

data spectra have high power at frequencies close to zero missed by both models, and the model spectra begin their ascent as the data spectra are initially declining.

Unlike the frequency specific RMSAEs described above, integrated RMSAE results are not invariant to linear filtering, since the integrated RMSAE is the ratio of two sums whose elements carry different weights depending on the choice of filter. For this reason, I provide four tables of these results. Tables I and III display overall integrated RMSAEs and business cycle band average RMSAEs for growth rates, while tables II and IV provide results for HP-filtered and exact bandpass filtered series. Each table covers five studies distinguished by the choice of which variable's error variance is minimized.[20]

The most striking results for the two first-difference tables are those located in the investment rows. Here we find confirmation that, across all studies, the sunspots investment series fails relative to the data, while the RBC model manages markedly better results. Comparing the investment rows between tables I and III, notice that the results are substantially worse for the all-frequency average than for the business cycle frequency average in both models. This feature, also present in the output and employment results, reflects the steadily rising RMSAEs above the business cycle band. Results for output and employment, overall and within the business cycle band, are close across frameworks. Overall, the animal spirits economy performs slightly better with respect to output, a consequence of its ability to generate a hump. However, the hump in its employment spectrum does not sufficiently offset the effects of excessive high frequency action, so that the RBC model, with less high frequency power, achieves slightly lower average RMSAEs. Within the consumption and wage rows, notice that the sunspots economy generally manages somewhat better results in both tables. This is because the RBC spectrum is shifted slightly left of the animal spirits spectrum, and its low frequency peaks exceed those of the sunspots model. Similar conclusions arise from a comparison of model performance in tables II and IV.

[20] For example, shaded regions of tables I and III correspond to the growth rate study where output is chosen as the primary variable.

The results above indicate that the sunspots framework is more successful than the benchmark in *producing business cycles*; the model generates a higher concentration of spectral power within the business cycle band, and produces a spectral shape more consistent with postwar U.S. data. However, this success is accompanied by excessive high frequency variation in employment, investment and output series, and too much investment volatility in all but the lowest frequencies.

To investigate the robustness of these findings, I introduce separate externality parameters for capital and labor. Results across a wide range of values reveal that, under pairs of externality parameters preserving the spectral hump, the model retains the problem features discussed above.[21] However, a further modification to include convex capital adjustment costs allows a substantial reduction in the economy's high frequency action and excess investment volatility while maintaining the baseline model's essential spectral shape. The measure of fit results for the investment series improve significantly in the presence of small adjustment costs, a small capital externality and a large labor externality. Tables 5a and 5b provide one such set of parameter choices and resulting integrated RMSAEs for growth rates and HP-filtered levels. Figure 7 displays the associated model spectra and RMSAEs for comparison with the original results of figures 3 and 6. In this and similar examples, spectral humps become more pronounced and shift further into the business cycle band, peaking at frequencies very close to those of the data's spectral peaks, and high frequency variability in output, employment and investment is much more consistent with the data. Such improvements may be of limited relevance, as the associated aggregate returns are in the region of 2; however, these initial explorations indicate that the combination of adjustment costs and separate capital and labor externalities may strengthen results for the sunspots framework.

[21] Parameter values yielding more substantial oscillations and spectral humps suffer the cost of a phase shift into the lower frequencies that removes spectral concentration from the business cycle range, and/or further exaggeration of the high frequency action in employment, output and investment.

6 Concluding Remarks

I conclude with a brief discussion of the generality of my findings. The current analysis has focussed on a comparison of the sunspots model of Farmer and Guo (1994) with the Hansen (1985) RBC model. This has been motivated by the controlled comparison allowed in considering a pair of reference models sharing nearly identical structures. We have seen that the sunspots model can better match the typical spectral shape of growth rates found in the data. This result is associated with the complex eigenvalues frequently obtained in animal spirits models. However, under certain parameterizations such models may be characterized by a pair of stable *real* eigenvalues, consistent with indeterminacy but lacking the data's spectral hump; hence this feature is not a foregone conclusion. We have also seen that the sunspots model suffers from excessive investment volatility and overstated high frequency action in employment, investment and output series.

Recent research has reduced the degree of increasing returns required to generate indeterminacy through the introduction of multiple sectors.[22] However, such models may have limited success in correcting the above mentioned difficulties. As long as propagation relies entirely upon movements in capital, the investment spectrum will continue to be large, and high frequency action will be excessive. Interaction across sectors, which acts to magnify the effects of increasing returns, should merely serve to produce these features under less extreme assumptions regarding the marginal product of labor and the level of returns in aggregate production. Efforts to reduce overstated investment volatility may lead researchers to concentrate on sunspots models containing multiple state variables, for instance those incorporating human capital accumulation, as in Benhabib and Perli (1994). The presence of an additional state variable relieves some of the pressure on investment to propagate the cycle, and hence should reduce the excessive investment spectrum. A quantitative evaluation of such models requires suitable time series on the new state variable.

[22] For example, see Benhabib and Farmer (1996) and Perli (1994).

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Appendix: Measures of Fit Technique

Below, I briefly review Watson's (Watson 1993) measures of fit technique, summarizing his benchmark model, the modifications necessary to apply the measure to business cycle models, and the calculations of the models' relative mean squared approximation errors.

.1 Benchmark Model

Represent the vector of data variables as y and the corresponding variables of the model as x, where x_t and y_t are assumed to be jointly covariance stationary. Next, define the individual autocovariance generating functions as $A_y(z)$ and $A_x(z)$, the joint autocovariance generating function as $A_{xy}(z)$, and the variance-covariance matrices as Σ_y, Σ_x, and Σ_{xy}. As Watson notes, there is neither a statistical nor theoretical basis for restrictions on the joint covariance properties of the model and data, since the error is neither sampling error nor forecast error and we have no knowledge of the underlying probability structure of the model. Watson's technique involves an unrestricted choice of Σ_{xy} or $A_{xy}(z)$ to yield the best possible model results.

Defining the model abstraction error as $u_t \equiv y_t - x_t$, the covariance matrix of the error is:

$$\Sigma_u = \Sigma_x + \Sigma_y - \Sigma_{xy} - \Sigma_{yx} \ . \tag{5}$$

In the case of nonsingular covariance matrices and no serial correlation, the benchmark case for minimum approximation error is given as the solution to

$$\min_{\Sigma_{xy}} \ (\ \mathrm{tr}\,[\Sigma_x + \Sigma_y - \Sigma_{xy} - \Sigma_{yx}]) \tag{6}$$

subject to the constraint that $\Sigma_{xy}(z)$ is positive semidefinite. We can also emphasize specific variables of the model by using a diagonal weighting matrix W, so that the problem minimizes the weighted trace of the error covariance matrix. Below I choose W to be the identity matrix I_n, and in this case the solution implies that x_t and y_t are perfectly correlated, with $x_t = \Gamma y_t$, so that Γ provides the matrix from which we can obtain fitted values for x, x_t^f, based on the data y_t. In particular, the solution sets

$$\Sigma_{xy} = C_x^{'} R^{'} C_y, \tag{7}$$

where C_x and C_y are "square root" solutions to $\Sigma_x = C_x^{'} C_x$ and $\Sigma_y = C_y^{'} C_y$, and R is the orthonormal matrix: $R = D\Delta^{-\frac{1}{2}} D^{'} C^{'}$, where $C = C_x W C_y^{'}$, Δ is the diagonal matrix of eigenvalues of C , and D is a matrix of orthonormal eigenvectors of $C^{'} C$. Since the solution implies that $\Sigma_{xy} = \Gamma \Sigma_y$, we have $\Gamma = C_x^{'} R^{'} C_y^{-1}$.

.2 Application of the technique to Sunspots and RBC Models

To apply Watson's technique to a business cycle model, we must use the version adapted to handle the singular covariance matrix and serial correlation structure implied by such models. While serial correlation makes the measure difficult to apply in the time domain, Watson's analysis carries over easily into the frequency domain. Here, we minimize the (weighted) trace of the spectral density matrix of u_t, where u_t, x_t, and y_t are broken into their Cramer representations; each variable is represented as the integral of complex valued increments that are uncorrelated across frequencies.

$$\begin{aligned} x_t &= \int_0^{2\pi} e^{i\omega t} dz_x(\omega) \\ y_t &= \int_0^{2\pi} e^{i\omega t} dz_y(\omega) \\ u_t &= \int_0^{2\pi} e^{i\omega t} dz_u(\omega) \end{aligned} \tag{8}$$

The spectral density matrix for abstraction error is given by $S_u(\omega) = \frac{1}{2\pi} A_u(e^{-i\omega})$, where i is the imaginary unit, and the autocovariance generating function for the error at $z = e^{-i\omega}$ is:

$$A_u(e^{-i\omega}) = A_y(e^{-i\omega}) + A_x(e^{-i\omega}) - A_{xy}(e^{-i\omega}) - A_{yx}(e^{-i\omega}) \,. \tag{9}$$

Since the increments $dz_u(\omega)$ are uncorrelated across frequencies, minimization of the variance of $dz_u(\omega)$ can be solved by minimizing the error's spectral density matrix independently frequency by frequency. For each ω_i, we solve

$$\min_{A_{xy}(e^{-i\omega_i})} \; tr\left[A_y(e^{-i\omega_i}) + A_x(e^{-i\omega_i}) - A_{xy}(e^{-i\omega_i}) - A_{yx}(e^{-i\omega_i})\right] \tag{10}$$

subject to the constraint that $A_{xy}(e^{-i\omega_i})$ is positive semidefinite, or, equivalently,

$$\max tr\left(A_{xy}(e^{-i\omega_i})\right) \text{ s.t. } \left|A_{xy}(e^{-i\omega_i})\right| \geq 0. \tag{11}$$

This is almost the problem that is solved to evaluate the RBC and sunspots models; however, given that there is only one shock process for each model, the maximization problem in (11) is not a feasible problem, so we must use a selection matrix S to reduce the number of variables considered in the error minimization problem to k, the rank of the model's spectral density matrix,[23] Thus, the relevant optimization problem when applying the technique to the sunspots and real business cycle frameworks is:

$$\max tr\left(A_{\widetilde{x}\widetilde{y}}(e^{-i\omega_i})\right) \text{ s.t. } \left|A_{\widetilde{x}\widetilde{y}}(e^{-i\omega_i})\right| \geq 0,$$

where $\widetilde{x_t} = Sx_t$ and $\widetilde{y_t} = Sy_t$. The solution to this problem is the complex analogue of the solution to the benchmark model, setting the cross spectrum as:

$$A_{xy}(e^{-i\omega}) = \Gamma(\omega)A_y(e^{-i\omega}) \tag{12}$$

where $\Gamma(\omega)$ is the complex analogue of $B\left(C'_{\widetilde{x}}\widetilde{R}'C^{-1}_{\widetilde{y}}\right)S$, with B being the $n \times k$ matrix used to retrieve the rank-constrained solution for the original vector of variables.[24] As in the benchmark model, this yields perfect correlation between $dz_x(\omega)$ and $dz_y(\omega)$,

$$dz_x(\omega) = \Gamma(\omega)dz_y(\omega). \tag{13}$$

.3 Approximation Error Calculations

After solving for the cross spectrum, we may obtain the minimum variance error spectrum as

$$S_u(\omega) = S_y(\omega) + S_x(\omega) - \Gamma(\omega)S_y(\omega) - (\Gamma(\omega)S_y(\omega))'. \tag{14}$$

[23] Here, $k = 1$, so that the selection matrix applied to the model leaves a simple scalar error minimization problem.

[24] Letting $\overline{S}$ be an $(n-k) \times n$ full row rank matrix of eigenvectors corresponding to the zero eigenvalues of Σ_x, B is composed of the first k columns of the matrix $\left[\left(S' \;\; \overline{S}'\right)'\right]^{-1}$.

Using this, with the estimated data spectra, we can calculate the relative mean squared approximation error (RMSAE) frequency by frequency for each variable i as:

$$\text{RMSAE}_i(\omega_j) = \frac{S_u(\omega_j)_{ii}}{S_d(\omega_j)_{ii}} \ , \tag{15}$$

where $S_d(\omega)_{ii}$ is the real component of the ith diagonal element of the estimated spectral density matrix of the data and $S_u(\omega)_{ii}$ is the real component of the ith diagonal element of the minimization error spectrum in (14). It is then straightforward to obtain the overall measure of relative mean squared approximation error, "integrated RMSAE," for each variable by calculating a weighted sum of the numerator and the denominator in (15) over all frequencies and then calculating the error to data ratio. Finally, this average RMSAE is provided for the business cycle band by calculating the integrated RMSAE over only the business cycle frequencies.

FIGURE 1: Impulse Responses for Sunspots Model*

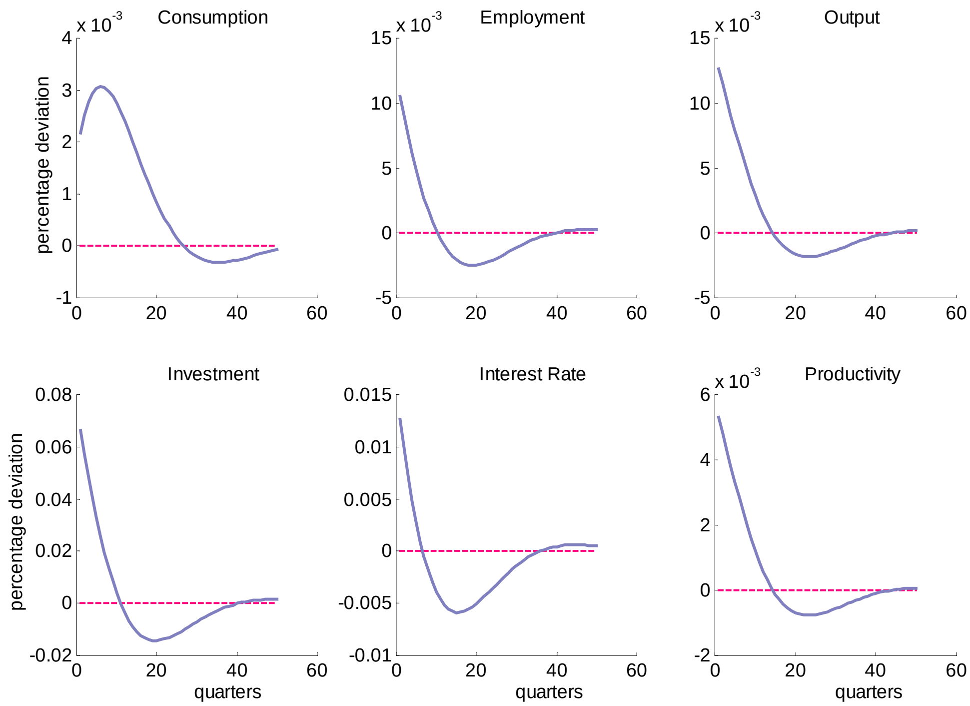
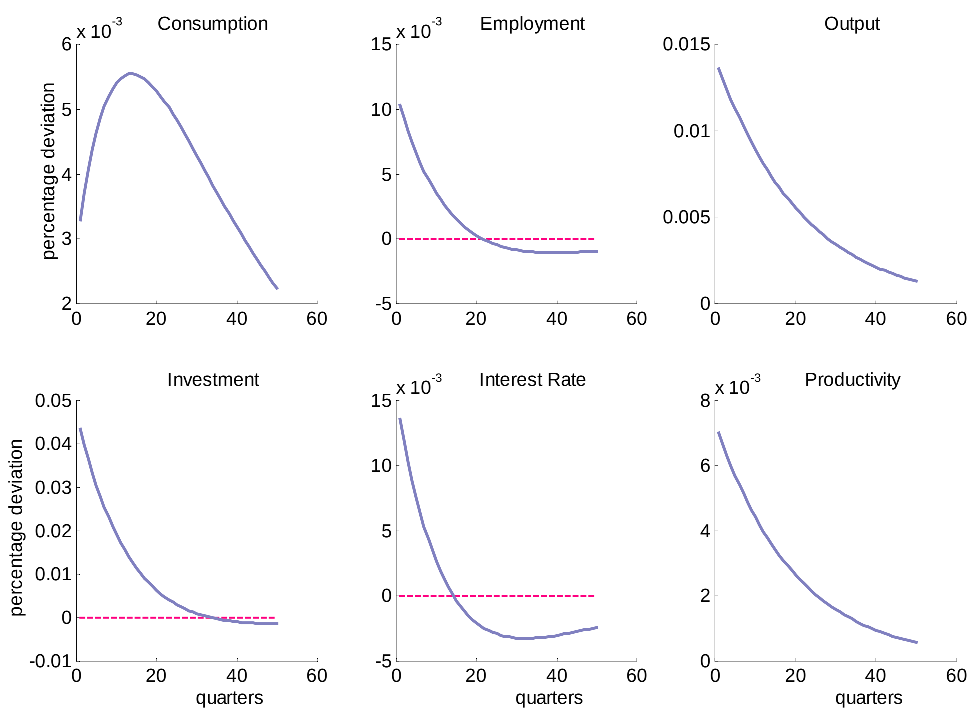
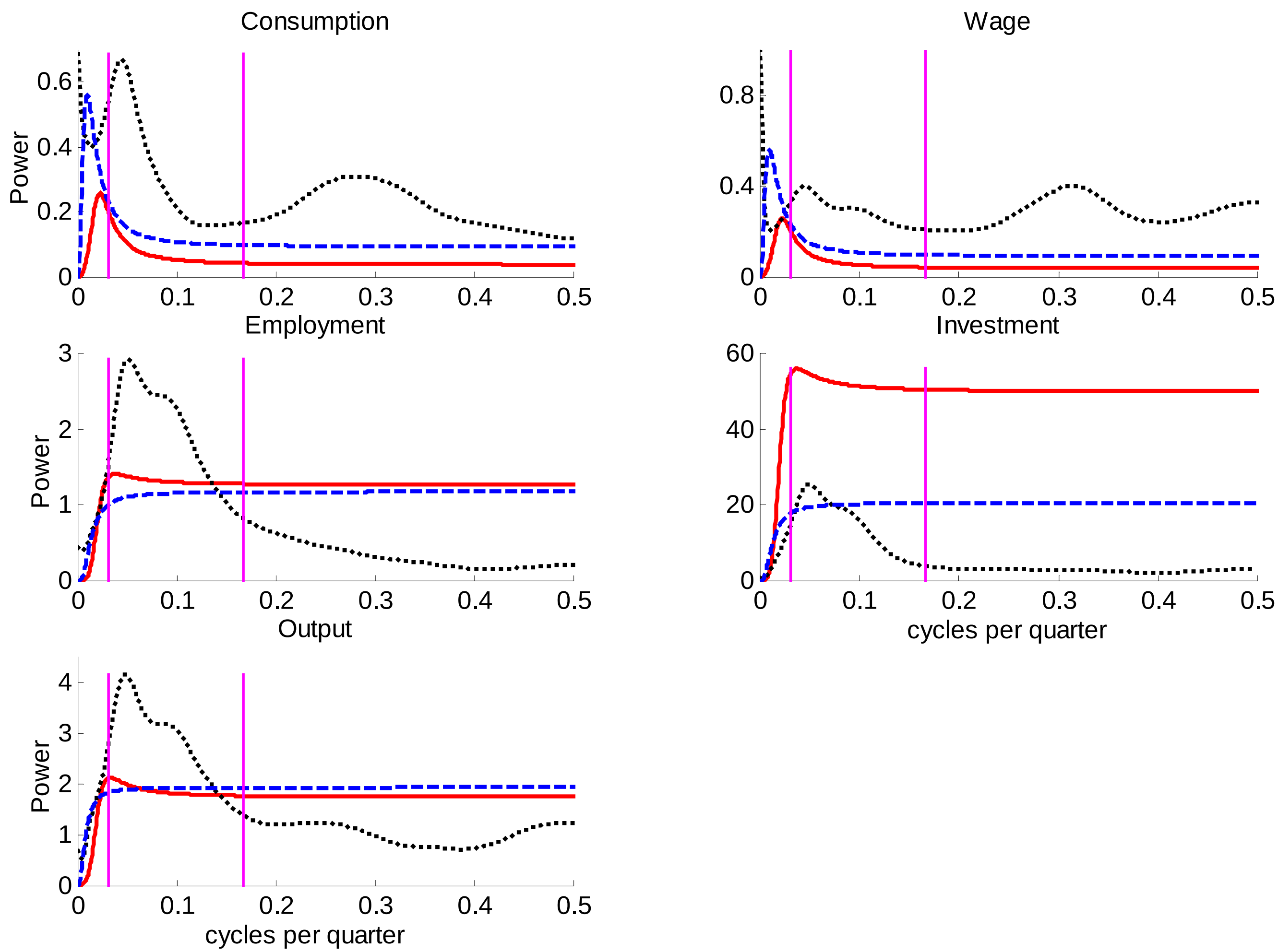


FIGURE 2: Impulse Responses for Real Business Cycle Model



* For both models, wage responses are identical to those of consumption.

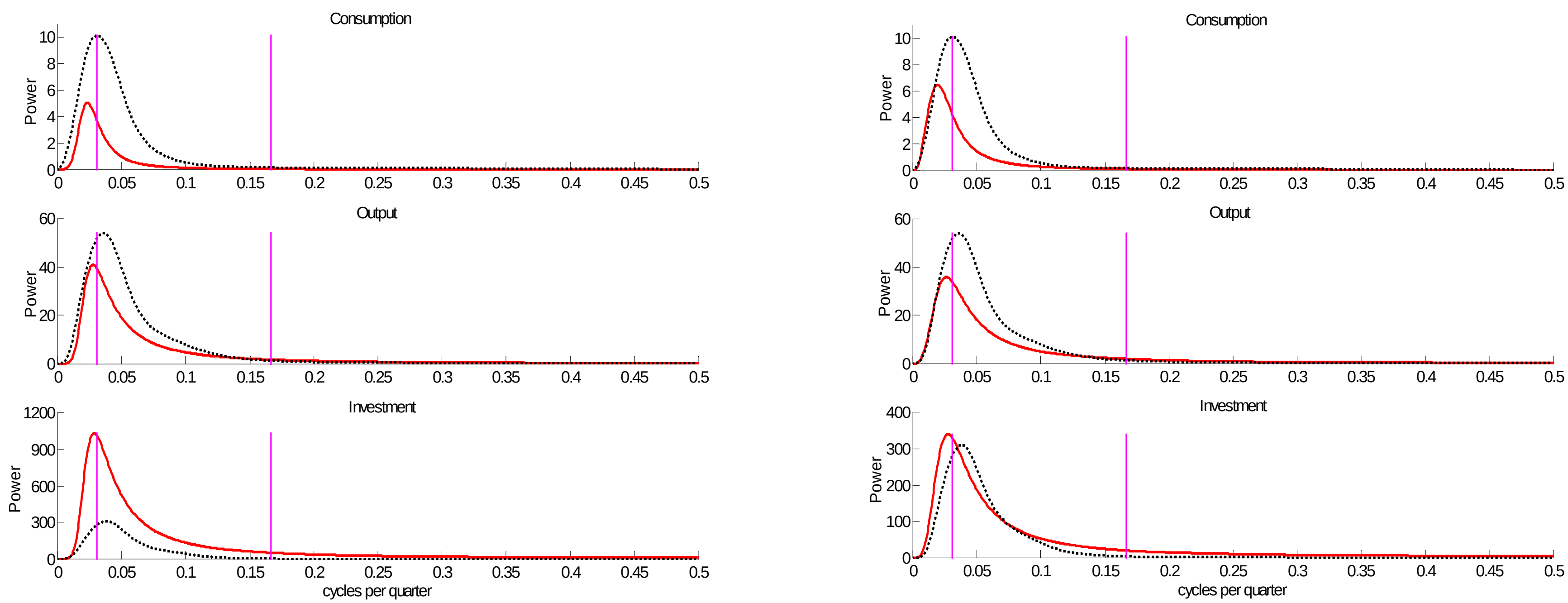
FIGURE 3: Growth Spectra for Data and Models *



* Solid (dashed) curves represent spectra for Sunpots (RBC) model. Dotted curves represent growth spectra for postwar quarterly U.S. data. Scaling is to percent; spectra do not integrate to variance over -Π to Π.

FIGURE 4: Spectra for HP-Filtered Series*

SUNSPOTS MODEL | REAL BUSINESS CYCLE MODEL

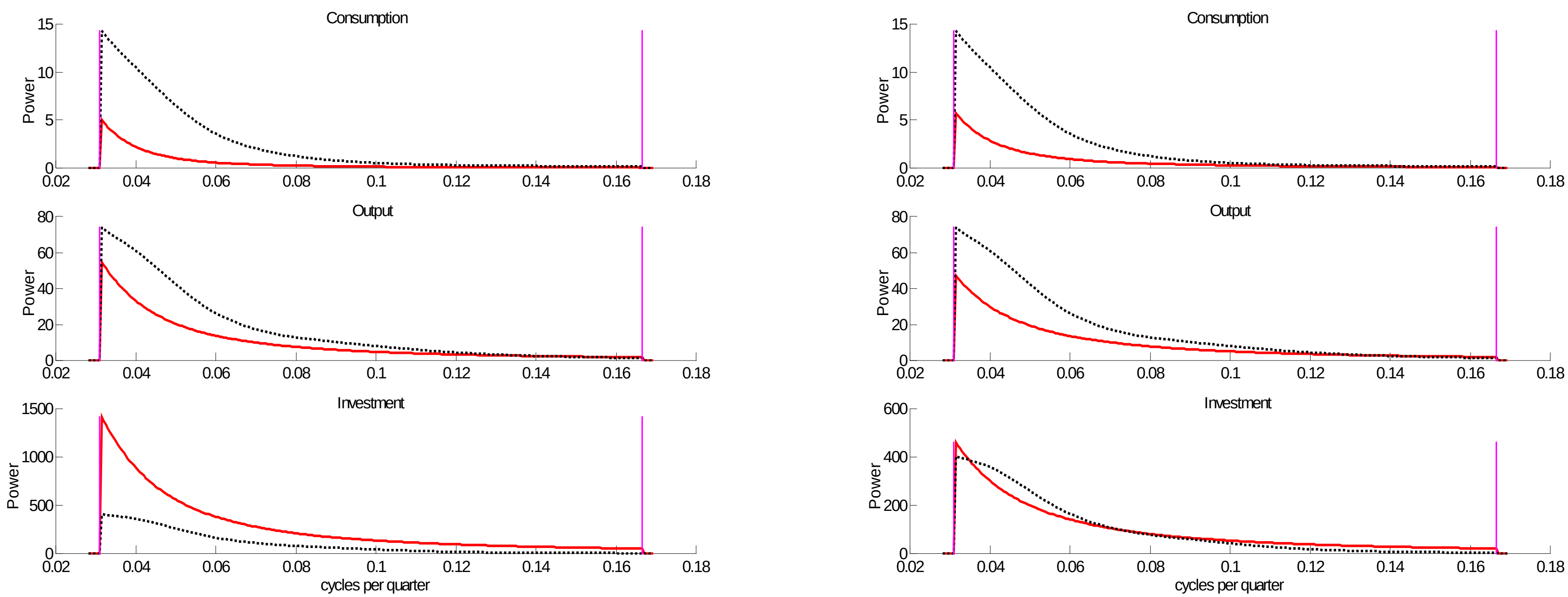


* Dotted curves represent spectra for HP-filtered postwar quarterly U.S. data. Scaling is to percent.

FIGURE 5: Spectra for Levels in the Business Cycle Band*

SUNSPOTS MODEL

REAL BUSINESS CYCLE MODEL



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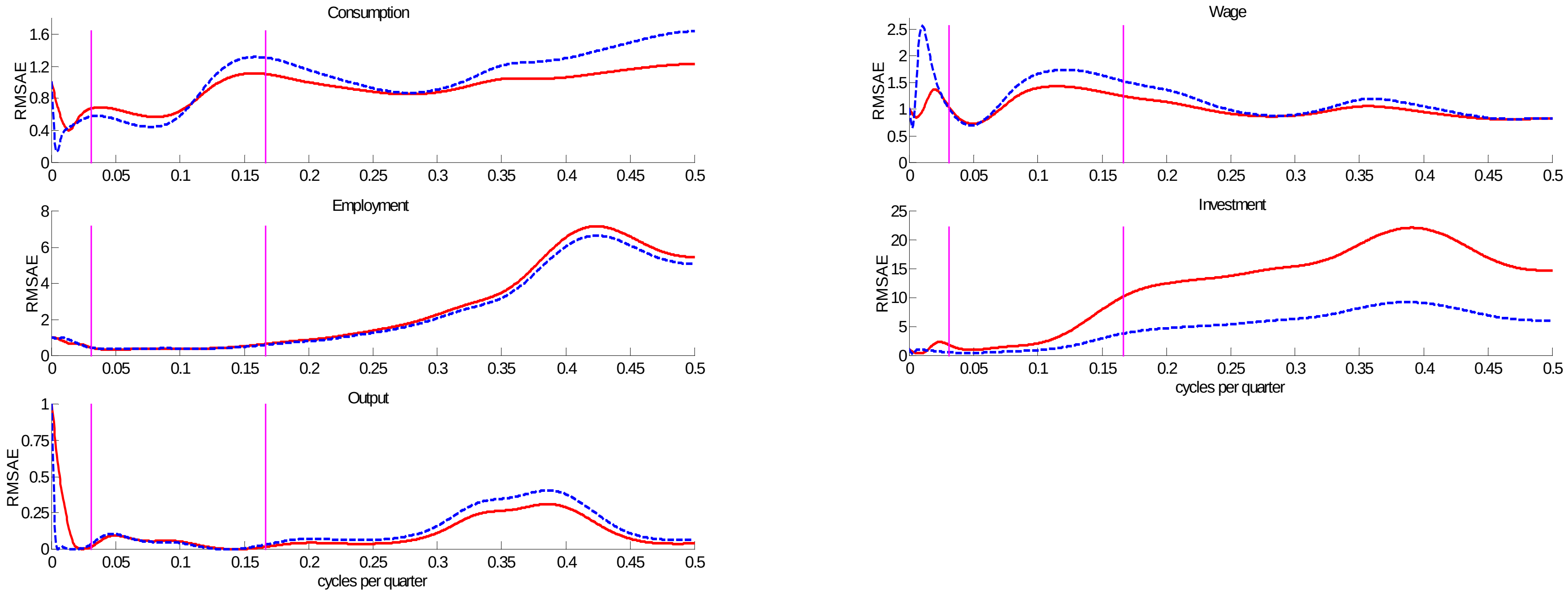
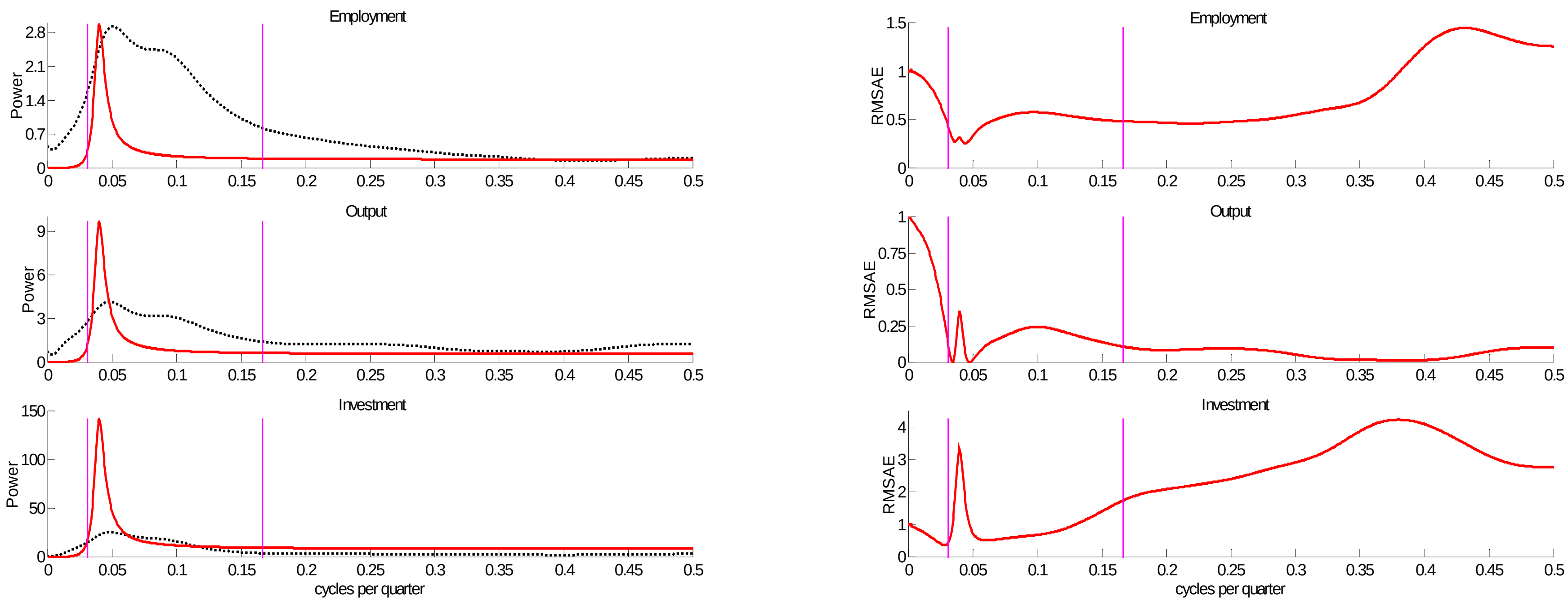
* Dotted curves represent spectra for band-pass filtered postwar quarterly U.S. data. Scaling is to percent.

FIGURE 6: Relative Mean Squared Approximation Errors*
(error minimized with respect to output)



* Solid (dashed) curves represent RMSAEs for Sunspots (RBC) model.

FIGURE 7: Growth Spectra and RMSAEs for Sunspots with Adjustment Costs*
(error minimized with respect to output)



* Dotted curves represent growth spectra for postwar quarterly U.S. data.

INTEGRATED RELATIVE MEAN SQUARE APPROXIMATION ERRORS

TABLE 1: First Differences: All Frequencies

minimization with respect to:

	Consumption		Employment		Output		Wage		Investment	
	S	RBC	S	RBC	S	RBC	S	RBC	S	RBC
Consumption	0.31	0.13	1.09	1.23	0.86	0.91	0.98	1.13	1.09	1.20
Employment	2.26	2.04	0.33	0.31	0.99	0.95	2.82	2.66	1.16	1.10
Output	1.41	1.42	0.77	0.83	0.08	0.09	1.75	1.81	1.16	1.22
Wage	1.00	1.14	1.28	1.55	1.01	1.13	0.36	0.17	1.22	1.44
Investment	8.25	3.63	5.78	2.24	6.38	2.60	9.20	4.37	3.78	0.96

TABLE 2: Levels: Business Cycle Frequencies

minimization with respect to:

	Consumption		Employment		Output		Wage		Investment	
	S	RBC	S	RBC	S	RBC	S	RBC	S	RBC
Consumption	0.29	0.20	1.09	1.03	0.67	0.57	0.76	0.79	1.15	1.10
Employment	1.34	1.11	0.06	0.11	0.37	0.39	1.55	1.40	0.20	0.23
Output	0.69	0.58	0.37	0.38	0.06	0.07	1.14	1.10	0.53	0.52
Wage	0.80	0.86	1.26	1.35	0.95	1.00	0.22	0.12	1.32	1.41
Investment	3.52	1.61	0.77	0.20	1.49	0.62	3.82	1.96	0.49	0.03

INTEGRATED RELATIVE MEAN SQUARE APPROXIMATION ERRORS

TABLE 3: First Differences: Business Cycle Frequencies

minimization with respect to:

	Consumption		Employment		Output		Wage		Investment	
	S	RBC	S	RBC	S	RBC	S	RBC	S	RBC
Consumption	0.29	0.16	1.09	1.12	0.72	0.67	0.95	1.06	1.16	1.20
Employment	1.44	1.25	0.06	0.08	0.39	0.40	1.89	1.78	0.31	0.32
Output	0.85	0.76	0.39	0.40	0.05	0.05	1.57	1.59	0.64	0.64
Wage	1.00	1.13	1.34	1.56	1.14	1.31	0.27	0.13	1.36	1.59
Investment	4.35	2.02	1.46	0.45	2.25	0.93	5.08	2.67	0.93	0.12

TABLE 4: Hodrick-Prescott Filtered Levels

minimization with respect to:

	Consumption		Employment		Output		Wage		Investment	
	S	RBC	S	RBC	S	RBC	S	RBC	S	RBC
Consumption	0.24	0.14	1.15	1.17	0.66	0.58	0.74	0.80	1.13	1.12
Employment	1.47	1.25	0.08	0.10	0.46	0.47	1.75	1.61	0.28	0.30
Output	0.75	0.65	0.44	0.47	0.06	0.06	1.21	1.19	0.58	0.57
Wage	0.81	0.93	1.47	1.71	1.03	1.16	0.19	0.11	1.46	1.68
Investment	3.98	1.80	1.26	0.38	1.96	0.78	4.49	2.29	0.84	0.11

TABLE 5:
EXAMPLE OF SUNSPOTS WITH INVESTMENT ADJUSTMENT COSTS

TABLE 5a: Parameters

γ	a	b	Ψ(K)	Ψ(N)	δ	ρ	σ	etastar
0.000	0.230	0.700	0.111	1.575	0.025	0.990	0.00046	0.100

Variance of the iid shock is chosen to match the standard deviation of HP-filtered U.S. output. Etastar is the elasticity of the marginal adjustment cost function. As in Baxter and Crucini (1993), the steady-state Tobin's q is 1, and the steady-state investment share matches that of the model without adjustment costs.

TABLE 5b: Integrated Rbounds for Growth Rates (GR) and Hodrick-Prescott Filtered Levels (HP)

minimization with respect to:

	Consumption		Employment		Output		Wage		Investment	
	GR	HP	GR	HP	GR	HP	GR	HP	GR	HP
Consumption	0.15	0.25	1.17	0.93	0.94	0.58	1.27	1.03	1.22	1.06
Employment	0.96	0.74	0.29	0.30	0.55	0.47	1.34	1.10	0.61	0.40
Output	0.87	0.53	0.55	0.46	0.15	0.21	1.20	1.07	0.81	0.60
Wage	1.26	1.30	1.62	1.79	1.21	1.40	0.18	0.32	1.55	1.88
Investment	2.14	1.68	1.21	0.66	1.51	1.05	2.95	2.45	0.34	0.42