

# Revisiting the Tale of Two Interest Rates with Endogenous Asset Market Segmentation

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## ABSTRACT

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We develop a monetary model that is unique in its ability to deliver a negative correlation between aggregate consumption growth and short-term real interest rates consistent with U.S. data. The essential ingredient to this success is endogenous asset market segmentation permitting the extent of household participation in asset markets to vary smoothly with changes in aggregate conditions. Households in our model incur fixed transactions costs when exchanging bonds and money and, as a result, carry money balances in excess of current spending to limit the frequency of such trades. While we impose no stickiness at the microeconomic level in either prices or portfolio adjustment, our model drives gradual adjustment in the aggregate price level following a monetary shock and thus persistent non-neutralities. In our model, households can alter the timing of their trading activities in response to changes in both individual and aggregate states. We show that this added flexibility relative to fixed segmentation models can substantially reinforce the sluggishness in aggregate price adjustment following a monetary shock, and it can transform dramatic, transitory changes in real and nominal interest rates into more moderate and persistent liquidity effects. When we extend our setting to consider production, we find that small changes in household participation rates add substantial persistence to movements in inflation, and they deliver persistence in real interest rates that is otherwise absent. These changes are also critically important to our model's success in reproducing the empirical correlation between aggregate consumption growth and real interest rates; when they are suppressed, the success is lost.

KEYWORDS: segmented markets; liquidity effects; (S,s) rules

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# 1 Introduction

There is a wealth of empirical research documenting co-movement between real and nominal series, and what is widely accepted as evidence of persistent responses in real variables following nominal disturbances. Towards better understanding such relationships, we develop a monetary model with endogenous asset market segmentation wherein heterogeneous households face fixed costs to shift their wealth between interest-bearing assets and money. Given these transactions costs, households in our model infrequently access their interest income, and they carry money balances in excess of current spending to help finance their spending over coming periods.

The most common approach to analyzing the relations between movements in real and nominal aggregate series is to use models with nominal prices that are sticky at the firm level and assume that firms must produce to satisfy demand at their given prices. These models have grown in sophistication with the inclusion of various aggregate frictions and stochastic driving processes, and they are widely used by central banks for monetary policy analysis.<sup>1</sup> However, there are still open issues confronting the sticky-price framework, beyond the well-known debate over the frequency of microeconomic price adjustment.<sup>2</sup> Below, we describe two such issues involving the model's predictions for the relation between short-term interest rates and real economic activity, each of which is resolved in our model without imposing nominal rigidity at the microeconomic or aggregate level.

First, because sticky price models assume a representative household, they have at their core a consumption Euler equation leading them to predict real interest rates tied to the growth rate of aggregate consumption. Canzoneri, Cumby and Diba (2007) show that this prediction is refuted in the data not simply in terms of average levels, but more critically in the directions of change. Using U.S. consumption data, Canzoneri et al. retrieve the interest rate series implied by the model's Euler equation under a series of leading preference specifications (including five commonly used variants of habit persistence). They find that the model-implied interest rate series is consistently negatively correlated with observed U.S. interest rates.

Second, without additional elements specifically designed to obtain it, the model has no inherent liquidity effect.<sup>3</sup> This issue arises irrespective of whether firm-level price stickiness is imposed or endogenous.<sup>4</sup> Dotsey and King (2005) resolve a long-standing issue for the sticky-price literature by developing an (S,s) model of nominal price setting that is consistent with empirical estimates of the persistence in inflation movements. In doing so, however, they find that the model predicts a rise in short-term nominal interest rates following a persistent positive shock to money growth rates.

Like other settings with segmented asset market participation, open market operations have real effects in our model economy because they directly involve only a subset of households, and because real balances are essential in facilitating goods market transactions. As we explain below, our model

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<sup>1</sup>Prominent examples are the models of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003).

<sup>2</sup>See Bils and Klenow (2004) and Nakamura and Steinsson (2008).

<sup>3</sup>Friedman (1968) highlights the liquidity effect as the starting point for the monetary transmission mechanism, noting “*The initial impact of increasing the quantity of money at a faster rate than it has been increasing is to make interest rates lower for a time than they would otherwise have been.*” VAR studies consistently uncover liquidity effects. See, for example, Leeper et al. (1996) or Christiano et al. (1996, 1999).

<sup>4</sup>This is a well known problem for the basic sticky price model; see, for example, King and Watson (1996) or Christiano et al. (1997). Christiano et al. (2005) add habit persistence in consumption and aggregate investment adjustment costs to create a liquidity effect. Edge (2007) shows the same result can be achieved through a combination of habit-persistence alongside time-to-build and time-to-plan in aggregate investment.

produces gradual adjustment in the general price level through a velocity mechanism associated with changes in the distribution of money holdings across periods.<sup>5</sup> This velocity mechanism is strengthened in our model by time variation in the fractions of households participating in the bond markets; as a result, it delivers greater persistence in inflation movements relative to models with exogenous market segmentation.

Our model has a natural liquidity effect. Because households participating in the bond markets at the date of an unanticipated rise in money growth carry unusually high real balances, they experience raised consumption relative to households participating in the bond markets in subsequent dates. This implies a fall in the real interest rate exceeding the rise in expected inflation. Unlike settings with time-invariant segmentation, however, the liquidity effect in our model is persistent, because endogenous changes in household participation rates limit the rise in inequality at such times. In particular, raised participation rates spread the rise in real balances out beyond the initial group of households to those households participating in the bond markets at later dates, thus generating a persistent rise in participating households' consumption.

Importantly, our model succeeds in reproducing the negative correlation between aggregate consumption growth and real interest rates observed in postwar U.S. data. As noted above, this is essentially impossible in any model that assumes a representative household. By contrast, real interest rates in our economy have no direct mapping to the aggregate consumption series; rather, they are determined by the relative marginal utilities of subsets of households participating in the bond markets at adjacent dates. Interestingly, however, the mere presence of asset market segmentation is not sufficient to deliver the empirical correlation between aggregate consumption growth and real interest rates. While exogenous segmentation moves the correlation further from 1, we show that it remains positive unless the extent of market segmentation is allowed to change over time in response to aggregate conditions.

Our work builds on an important literature that studies monetary policy in models with exogenously segmented markets.<sup>6</sup> We contribute to the segmented markets literature what the menu cost model contributes to the sticky-price literature. We endogenize the microeconomic stickiness at the heart of the model's non-neutralities - in this case, the infrequency of household portfolio adjustments.

The model we develop is similar to the work of Grossman and Weiss (1983), Rotemberg (1984), and Alvarez, Atkeson and Edmond (2009) in that the households therein only periodically access the market for interest-bearing bonds (broadly interpreted as markets for relatively high-yield, illiquid assets).<sup>7</sup> As a result, they carry inventories of money (interpreted as relatively low yield, liquid assets) to help finance their spending over several periods of future spending. This allows aggregate velocity to vary over time, and thus permits sluggish adjustment in the price level even in an endowment economy setting.

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<sup>5</sup>Many VAR studies suggest that the general price level adjusts slowly following nominal shocks. See, for example, Leeper, Sims, and Zha (1996), Christiano, Eichenbaum, and Evans (1999), and Uhlig (2005). King and Watson (1996) also show that the price level is positively correlated with lagged real output at business cycle frequencies. Further evidence of gradual aggregate price adjustment may be seen in the relation of short-term movements in velocity to those in the ratio of money to consumption, as is discussed by Alvarez, Atkeson and Edmond (2009). The correlation between the ratio of money (M2) to consumption (PCE) and the corresponding measure of velocity is -0.89 for HP-filtered monthly data.

<sup>6</sup>See Alvarez, Lucas and Weber (2001) and the references therein.

<sup>7</sup>A second branch of the exogenously segmented markets literature assumes a fixed subset of households is always participating in asset markets, while a second subset is permanently excluded. Recent examples include Occhino (2004, 2008) and Williamson (2008).

In our model, as in the inventory-theoretic model of Alvarez, Atkeson and Edmond (2009), household money spending rates are lowest among households that have recently transferred wealth held as bonds into money, and they rise with the time since such a transfer has occurred. In such an environment, changes in monetary policy alter the distribution of money holding across households with different spending rates, and this, in turn, has implications for interest rates and inflation. More specifically, an unanticipated expansionary open market operation increases the relative money holdings of households currently trading bonds for money. Because such households have the lowest money spending rates across all households, this leads to a fall in the aggregate spending rate, or aggregate velocity. Thus, nominal spending rises by less than the money supply, delaying the rise in the aggregate price level. Moreover, as noted above, a liquidity effect emerges directly from the fact that households trading in the bond markets at the date of the shock see the greatest rise in their consumption, since real interest rates are determined by the consumption of participating households at adjacent dates.

Our model is distinguished from Alvarez, Atkeson and Edmond (2009) and other monetary models with segmented asset markets by the fact that the fraction and composition of households directly involved in an open market operation is endogenously determined. Households in our economy individually choose when to adjust their portfolios of bonds and money by choosing when to pay fixed transactions costs required for such exchanges, and they face idiosyncratic risk influencing the timing of these activities. In this respect, we build on the work of Alvarez, Atkeson and Kehoe (2002). Our work here offers an independent theoretical contribution in formally establishing how their results can be extended to a setting with persistent heterogeneity.<sup>8</sup> Because our households do not typically exhaust their money balances in current consumption expenditure, we have a nontrivial distribution of money holdings evolving across periods, which permits the changes in aggregate velocity that give rise to sluggish adjustment in the aggregate price level. We ensure that differences across our households are persistent, but not permanent by assuming a full set of state-contingent nominal bonds permitting risk-sharing.

Our model is unique in incorporating an evolving distribution of money in a setting where the extent of asset market segmentation varies smoothly over time with changes in the aggregate state. By including idiosyncratic differences across households, we ensure that the fraction of households choosing to participate in the asset markets remains nontrivial over time, as does the distribution of money. This has important implications for the propagation of aggregate disturbances. In particular, we show that modest changes in participation rates gradualize aggregate inflation movements relative to those in a model with time-invariant market segmentation, and they transform sharp, temporary movements in real and nominal interest rates into more moderate, persistent ones.<sup>9</sup> Finally, as noted above, these changes are essential to our model's unique success in reproducing the empirical correlation between aggregate consumption growth and real interest rates.

The presence of state-contingent bonds in our economy does not lead to full insurance, because households must pay fixed transactions costs to access their bond holdings. Households that are ex-ante identical diverge over time as idiosyncratic realizations of shocks drive differences in their money and bond holdings. Nonetheless, whenever a heterogeneous group of households enters the bond market at the same time, we prove that all previous differences between them are eliminated

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<sup>8</sup>Chatterjee and Corbae (1991) also study an economy where some households pay a fixed cost to trade bonds.

<sup>9</sup>One exogenous segmentation model that also succeeds in this respect is the model of Williamson (2008) wherein households are permanently divided into groups with and without access to the asset markets. There, the assumption that households with (without) such access prefer to trade among themselves in the goods markets delivers a second type of segmentation that can lead to persistent liquidity effects.

so that our model economy exhibits limited memory. Exploiting this property, we show how the numerical approach to solving generalized (S,s) models developed by King and Thomas (2006) can be applied in a setting where the consumption and savings decisions of heterogeneous risk-averse households are directly influenced by nonconvex costs. While the approach has been adopted elsewhere in solving models where risk-neutral production units face idiosyncratic fixed costs of adjusting their prices or factors of production (as in Dotsey, King and Wolman (1999) and Thomas (2002)), this is to our knowledge the first application involving heterogeneity among households.

## 2 An endowment economy

We begin by considering an endowment economy. We explore this setting at some length to clearly illustrate the mechanics of endogenous segmentation most central to our model. The full model with production and its results will be described in section 4. Below, we provide an overview of the endowment model. Thereafter, we proceed to a more formal description of households' problems, followed by the description of a financial intermediary that sells households claims contingent on both aggregate and individual states. Next, we show that there is an equivalent, but more tractable, representation of households' lifetime optimization problems, given their ability to purchase such individual-state-contingent bonds alongside the fact that they are ex-ante identical. Proofs of all lemmas are provided in the appendix.

### 2.1 Overview

The model economy has three sets of agents: a unit measure of ex-ante identical households, a perfectly competitive financial intermediary, and a monetary authority. Each infinitely-lived household values consumption in every date of life, with period utility  $u(c)$ , and it discounts future utility with the constant discount factor  $\beta$ , where  $\beta \in (0, 1)$ . In each period, households receive a common endowment,  $y$ . Since we will include both real and monetary shocks in our production economy below, we here allow for the possibility that the common endowment varies exogenously over time, as does the growth rate of the aggregate money supply,  $\mu$ . Defining the date  $t$  realization of aggregate shocks as  $s_t = (y_t, \mu_t)$ , we denote the history of aggregate shocks by  $s^t = (s_1, \dots, s_t)$ , and the initial-period probability density over aggregate histories by  $g(s^t)$ .

Households have two means of saving. First, they have access to a complete set of state-contingent nominal bonds. These are purchased from a financial intermediary described below, and are maintained in interest-bearing accounts that we will refer to as households' *brokerage accounts*, following Alvarez, Atkeson and Edmond (2009). Next, they also save using money, which they maintain in their *bank accounts* and use to conduct trades in the goods market.<sup>10</sup> Households have the opportunity to transfer assets between their two accounts at the start of each period; this occurs after the realization of all current shocks, but prior to any trading in the goods market. As such, it is expositionally convenient to refer to each period as consisting of two subperiods that we will term *transfer-time* and *shopping-time*, although nothing in the environment necessitates this approach.

There are three inter-related frictions leading households to maintain money in their bank accounts. First, as in a standard cash-in-advance environment, households cannot consume their

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<sup>10</sup>When allowed to store money in their brokerage accounts, households never do so given positive nominal interest rates paid on bonds. Thus, we simplify the model's exposition here by assuming that money is held only in bank accounts and verify that nominal rates remain positive throughout our results.

own endowments. Each household consists of a worker and a shopper, and the worker must trade the household endowment for money while the shopper is purchasing consumption goods. As a result, the household receives the nominal value of its endowment,  $P(s^t)y(s^t)$ , only at the end of the period after current goods trade has ceased. We assume that these end-of-period nominal receipts are deposited across their two accounts, with fraction  $\lambda$  paid into bank accounts and the remainder into brokerage accounts. Second, as all trades in the goods market are conducted with money, each household's consumption purchases are constrained by the bank account balance it holds when shopping-time begins.

Note that, absent other frictions, each household would, in every period, simply shift from its brokerage account into its bank account exactly the money needed to finance current consumption expenditure not covered by the bank account paycheck from the previous period. There is, however, a third friction that prevents this, leading households to deliberately carry money across periods; this is the assumption that they must pay fixed costs each time they transfer assets between their two accounts. Given these fixed costs, households maintain stocks of money to limit the frequency of their transfers, and they follow generalized (S,s) rules in managing their bank accounts.

Transfer costs are fixed in that they are independent of the size of the transfer; however, they vary over time and across households. Here, we subsume the idiosyncratic features that distinguish households directly in their fixed costs by assuming that each household draws its own current transfer cost,  $\xi$ , from a time-invariant distribution  $H(\xi)$  at the start of each period. Because this cost draw influences a household's decision of whether to undertake any transfer, and hence its current consumption and money savings, each household is distinguished by its history of such draws,  $\xi^t = (\xi_1, \dots, \xi_t)$ , with associated density  $h(\xi^t) = h(\xi_1) \cdots h(\xi_t)$ . As will be seen below, households are able to insure themselves in their brokerage accounts through the purchase of nominal bonds contingent on both aggregate and individual exogenous states.

## 2.2 Households

At the start of any period, given date-event history  $(s^t, \xi^t)$ , a household's brokerage account assets include nominal bonds,  $B(s^t, \xi^t)$ , purchased in the previous period at price  $q(s^t, \xi^t)$ , as well as the fraction of its income from the previous period that is deposited there,  $(1-\lambda)P(s^{t-1})y(s^{t-1})$ . The remainder, the paycheck,  $\lambda P(s^{t-1})y(s^{t-1})$ , is deposited into the household's bank account and supplements its money savings there from the previous period,  $A(s^{t-1}, \xi^{t-1})$ . Given this start of period portfolio and its current fixed cost, the household begins the period by determining whether or not to transfer assets across its two accounts. Denoting the household's start-of-period bank balance by  $M(s^{t-1}, \xi^{t-1})$ , where

$$M(s^{t-1}, \xi^{t-1}) \equiv A(s^{t-1}, \xi^{t-1}) + \lambda P(s^{t-1})y(s^{t-1}), \quad (1)$$

the relevant features of this choice are summarized in the chart below.

	brokerage account withdrawal	shopping-time bank balance
$z(s^t, \xi^t) = 1$	$x(s^t, \xi^t) + P(s^t)\xi_t$	$M(s^{t-1}, \xi^{t-1}) + x(s^t, \xi^t)$
$z(s^t, \xi^t) = 0$	0	$M(s^{t-1}, \xi^{t-1})$

An *active* household is indicated by  $z(s^t, \xi^t) = 1$ . In this case, the household selects a nonzero nominal transfer  $x(s^t, \xi^t)$  from its brokerage account into its bank account and has  $M(s^{t-1}, \xi^{t-1}) + x(s^t, \xi^t)$  available in its bank account at the start of the current shopping subperiod. Here, the

household's current fixed cost applies, so  $P(s^t)\xi_t$  is deducted from its nominal brokerage wealth. Alternatively, the household may choose to undertake no such transfer, setting  $z(s^t, \xi^t) = 0$  and remaining *inactive*. In that case, it enters into the shopping subperiod with no change to its start-of-period bank and brokerage account balances.

We assume that the period utility function,  $u(c)$ , is strictly increasing, strictly concave and twice-continuously differentiable, and that  $\lim_{c \rightarrow 0} u'(c) = \infty$ . Each household chooses its state-contingent plan for the timing and size of its account transfers ( $z(s^t, \xi^t)$  and  $x(s^t, \xi^t)$ ), and its bond purchases, money savings and consumption ( $B(s^t, s_{t+1}, \xi^t, \xi_{t+1})$ ,  $A(s^t, \xi^t)$  and  $c(s^t, \xi^t)$ ), to maximize its expected discounted lifetime utility,

$$\sum_{t=1}^{\infty} \beta^{t-1} \int_{s^t} \int_{\xi^t} u(c(s^t, \xi^t)) h(\xi^t) g(s^t) d\xi^t ds^t, \quad (2)$$

subject to the sequence of constraints in (3) - (6).

$$B(s^t, \xi^t) + (1 - \lambda) P(s^{t-1}) y(s^{t-1}) \geq \left[ x(s^t, \xi^t) + P(s^t) \xi_t \right] z(s^t, \xi^t) \quad (3)$$

$$+ \int_{s_{t+1}} \int_{\xi_{t+1}} q(s^t, s_{t+1}, \xi_{t+1}) B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) ds_{t+1} d\xi_{t+1}$$

$$M(s^{t-1}, \xi^{t-1}) + x(s^t, \xi^t) z(s^t, \xi^t) \geq P(s^t) c(s^t, \xi^t) + A(s^t, \xi^t) \quad (4)$$

$$A(s^t, \xi^t) + \lambda P(s^t) y(s^t) \geq M(s^t, \xi^t) \quad (5)$$

$$A(s^t, \xi^t) \geq 0 \quad (6)$$

Equation 3 is the household's brokerage account budget constraint associated with history  $(s^t, \xi^t)$ , and requires that expenditures on new bonds together with any transfer to the bank account and associated fixed cost not exceed current brokerage account wealth. Next, the bank account budget constraint in equation 4 requires that the household's money balances entering the shopping subperiod cover its current consumption expenditure and any money savings for next period. Money balances for next period, in (5), are these savings together with the bank paycheck received after completion of current goods trade. Equation 6 prevents the household from ending current trade with a negative bank balance; thus, taken together with the restriction in (4), it requires that cash be used for consumption purchases. Finally, in addition to this sequence of constraints, we also impose a standard limit condition on household debt:

$$\lim_{t \rightarrow \infty} \int_{s^t} \int_{\xi^t} q(s^t, \xi^t) B(s^t, \xi^t) ds^t d\xi^t \geq 0. \quad (7)$$

Following the approach of Alvarez, Atkeson and Kehoe (2002), we find it convenient to model risk-sharing by assuming a perfectly competitive financial intermediary that purchases government bonds with payoffs contingent on the aggregate shock and, in turn, sells to households bonds with payoffs contingent on both the aggregate and individual shocks. In particular, given aggregate history  $s^t$ , the intermediary purchases government-issued contingent claims  $B(s^t, s_{t+1})$  at price  $q(s^t, s_{t+1})$ , and it sells them across households as claims contingent on individual transfer costs,

$\xi_{t+1}$ . Note that, as households' cost draws are not autocorrelated, the price of any such claim is  $q(s^t, s_{t+1}, \xi_{t+1})$ , independent of the individual history  $\xi^t$ .

For each  $(s^t, s_{t+1})$ , the intermediary selects its aggregate bond purchases,  $B(s^t, s_{t+1})$ , and individual bond sales,  $B(s^t, s_{t+1}, \xi^t, \xi_{t+1})$ , to solve

$$\max_{\xi_{t+1}} \int \int_{\xi^t} q(s^t, s_{t+1}, \xi_{t+1}) B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi^t) d\xi^t d\xi_{t+1} - q(s^t, s_{t+1}) B(s^t, s_{t+1}) \quad (8)$$

subject to:

$$B(s^t, s_{t+1}) \geq \int \int_{\xi_{t+1}} B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi^t) h(\xi_{t+1}) d\xi^t d\xi_{t+1}. \quad (9)$$

The constraint in (9) requires that, for any  $(s^t, s_{t+1})$ , the intermediary must purchase sufficient aggregate bonds to cover all individual bonds held against it for that aggregate history. Given  $s_{t+1}$  occurs, fraction  $h(\xi_{t+1})$  of the households with history  $\xi^t$  to whom it sells such bonds will realize that state and demand payment. As shown in Lemma 1 below, the financial intermediary's zero profit condition immediately implies that the price of any individual bond associated with  $(s_{t+1}, \xi_{t+1})$  is simply the product of the price of the relevant aggregate bond and the probability of an individual household drawing the transfer cost  $\xi_{t+1}$ .

**Lemma 1.** *The equilibrium price of state-contingent bonds issued by the financial intermediary,  $q(s^t, s_{t+1}, \xi_{t+1})$ , is given by  $q(s^t, s_{t+1}, \xi_{t+1}) = q(s^t, s_{t+1}) h(\xi_{t+1})$ .*

By assuming an initial period 0 throughout which households are identical, we allow them the opportunity to trade in individual-state-contingent bonds at a time when they have the same wealth and face the same probability distribution over all future individual histories. In this initial period, the government has some outstanding debt,  $\bar{B}$ , that is evenly distributed across households' brokerage accounts, and it repays this debt entirely by issuing new bonds. Households receive no endowment, draw no transfer costs and do not value consumption in this initial period. Rather, they simply purchase state-contingent bonds for period 1 subject to the common initial period brokerage budget constraint:

$$\bar{B} \geq \int_{s_1} \int_{\xi_1} B(s_1, \xi_1) q(s_1) h(\xi_1) d\xi_1 ds_1.$$

Following the proof of Lemma 1, section B of the appendix shows that the period 0 budget constraint above can be combined with the sequence of constraints in (3) to yield the following lifetime budget constraint common to all households.

$$\bar{B} \geq \sum_{t=1}^{\infty} \int \int q(s^t) h(\xi^t) \left( z(s^t, \xi^t) \left[ x(s^t, \xi^t) + P(s^t) \xi_t \right] - P(s^{t-1}) (1 - \lambda) y(s^{t-1}) \right) d\xi^t ds^t, \quad (10)$$

where  $q(s^t) \equiv q(s_1) \cdot q(s_1, s_2) \cdots q(s^{t-1}, s_t)$ .

Finally, we assume that the monetary authority is subject to the sequence of constraints,

$$B(s^t) - \int_{s_{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}) ds_{t+1} = \bar{M}(s^t) - \bar{M}(s^{t-1}), \quad (11)$$



requiring that its current bonds be covered by a combination of new bond sales and the printing of new money. This sequence of constraints, alongside equilibrium in the money market, immediately implies that households' aggregate expenditures on new bonds in any period is exactly the difference between the aggregate of their current bonds and the change in the aggregate money supply:

$$\overline{M}(s^t) - \overline{M}(s^{t-1}) = B(s^t) - \int \int \int q(s^t, s_{t+1}) h(\xi_{t+1}) B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi^t) d\xi^t d\xi_{t+1} ds_{t+1}. \quad (12)$$

### 2.3 A risk sharing arrangement

Three aspects of the environment described above may be exploited to simplify our solution for competitive equilibrium: (i) fixed transfer costs are independently and identically distributed across households and time, (ii) households have perpetual access to a complete set of state-contingent claims in their brokerage accounts, and (iii) they are able to purchase these state-contingent bonds during an initial period in which they are perfectly identical. In this section, we show how these assumptions permit a more convenient representation of households' problems. In particular, exploiting the common lifetime budget constraint in (10) above, we will move from the household problem stated in section 2.2 to construct the equivalent problem of an extended family that manages all households' bonds in a joint brokerage account, and whose period-by-period decisions regarding bond purchases and account transfers implement the state-contingent lifetime plan selected by every household. In doing so, we transform our somewhat intractable initial problem into something to which we can apply the King and Thomas (2006) approach for solving aggregate economies involving heterogeneity arising due to (S,s) policies at the individual level.

***Money as the individual state variable:*** A complete set of state-contingent claims in the brokerage account allows individuals to insure their bond holdings against idiosyncratic risk; these shocks only affect their bank accounts. Alternatively, an individual's money balance fully captures the cumulative effect of his history of idiosyncratic shocks. In Lemma 2, we prove that, prior to their current transfer cost draws, all relevant differences across households are fully summarized by their start-of-period money balances as they enter into any period.

**Lemma 2.** *Given  $M(s^{t-1}, \xi^{t-1})$ , the decisions  $c(s^t, \xi^t)$ ,  $A(s^t, \xi^t)$ ,  $x(s^t, \xi^t)$  and  $z(s^t, \xi^t)$  are independent of the history  $\xi^{t-1}$ .*

Since each  $\xi$  comes from an i.i.d. distribution, a household's draw in any given period does not predict its future draws, and thus directly affects only its asset transfer decision in that one period. This certainly affects current shopping-time money balances, and hence consumption. However, its only future effect is in determining the money balances with which the household will enter the subsequent period, given the household's ability to insure itself in its brokerage account by purchasing bonds contingent on both aggregate and individual shocks. In proving this result, we show that the solution to the original household problem from section 2.2, given the lifetime constraint in (10), is identical to the solution of an alternative problem where households pool risk period-by-period by each committing to pay the economywide average of the total transfers and associated fixed costs incurred across all active households in every period, irrespective of the timing and size of their own portfolio adjustments. It is immediate from this that households' bond holdings may be modeled as independent of their individual histories  $\xi^t$ . Thus, within every

period, the distinguishing features affecting any household's decisions can be summarized entirely by its start-of-period bank balance,  $M(s^{t-1}, \xi^{t-1})$ , and its current transfer cost,  $\xi_t$ .

**Households as members of time-since-active groups:** Our next lemma establishes that, within any period, all households that undertake an account transfer will select both a common consumption and a common end-of-period bank balance; hence they begin the subsequent period with the same bank (and brokerage) account balances.

**Lemma 3.** *For any  $(s^t, \xi^t)$  in which  $z(s^t, \xi^t) = 1$ ,  $c(s^t, \xi^t)$ ,  $A(s^t, \xi^t)$  and  $M(s^t, \xi^t)$  are independent of  $\xi^t$ .*

To understand this result, recall that household brokerage and bank accounts are joined in periods when they choose to adjust their portfolios, and all are identical when they make their state-contingent plans in date 0. Given this, in selecting their consumption for such periods, households equate their appropriately discounted marginal utility of consumption to the multiplier on the lifetime brokerage budget constraint from (10), which is common to all households. Next, in selecting what portion of their shopping-time bank balances to retain after consumption (hence their next-period balances), households equate the marginal utility of their current consumption to the expected return on a dollar saved for the next period weighted by their expected discounted marginal utility of next-period consumption. Given common inflation expectations and the common current consumption of active households, this implies that such households also share in common the same expected consumption for next period. Thus, all currently active households exit this period and enter the next period with common money holdings.

Note that Lemmas 2 - 3 combine to imply that, within any period, households that undertake balance transfers all enter shopping-time with the same bank balance, make the same shopping-time decisions, and then enter the next period as effectively identical. Moreover, of this group of currently active households, those households that do not undertake an account transfer again in the next period will continue to be indistinguishable from one another as they enter shopping-time, and hence will enter the subsequent period with common bank (and brokerage) account balances, and so forth. In other words, any household that was last active at some date  $t$  is effectively identical to any other household last active at that same date. This is useful in our numerical approach to solving for competitive equilibrium, since it allows us to move from identifying individual households by their current money holdings to instead identifying each household as a member of a particular time-since-active group, with all members of any one such group sharing in common the same start-of-period money balances.

Given the above results, we may track the distribution of households over time through two vectors, one indicating the measures of households entering the period in each time-since-active group,  $[\theta_{j,t}]$ ,  $j = 1, 2, \dots$ , and the other storing the balances with which members of each of these current groups exited shopping-time in the previous period,  $[A_{j,t}]$ . From the latter, the current start-of-period balances held by members of each group are retrieved as  $M_{jt} = A_{j,t} + \lambda P_{t-1} y_{t-1}$ , where  $P_{t-1}$  represents the previous period's price level, and  $y_{t-1}$  the common endowment of the previous period. Households within any given start-of-period group  $j$  that do not pay their fixed costs move together into the current shopping subperiod with their starting balances  $M_{jt}$ . Across all start-of-period groups, those households that do pay to undertake a bank transfer will enter the current shopping subperiod in time-since-active group 0 with common shopping-time balances,  $M_{0,t}$ , which we refer to as the current *target* money balances.

**Threshold transfer rules:** Finally, we establish that households follow threshold policies in determining whether or not to transfer assets between their brokerage and bank accounts. Specifically, given its start-of-period money balances, each household has some maximum fixed cost that it is willing to pay to undertake an account transfer and adjust its balances to the current target.

**Lemma 4.** For any  $(s^t, \xi^{t-1})$ ,  $\mathcal{Z} = \{\xi_t \mid z(s^t, \xi^t) = 1\}$  is a convex set bounded below by 0.

As our preceding results imply that all members of any given start-of-period group  $j$  are effectively identical prior to the draws of their current transfer costs, this last result allows convenient determination of the fractions of each such group undertaking account transfers, and thus the shopping-time distribution of households. Define the threshold cost  $\xi_{jt}^T$  as that fixed cost that leaves any household in time-since-active group  $j$  indifferent to an account transfer at date  $t$ . Households in the group drawing costs at or below  $\xi_{jt}^T$  pay to adjust their portfolios, while other members of the group do not. Thus, within each group  $j$ , the fraction of its members shifting assets to reach the current target bank balance is given by  $\alpha_{jt} \equiv H(\xi_{jt}^T)$ . Each such active household undertakes a transfer  $x_{jt} = M_{0,t} - M_{j,t}$ , and total transfer costs paid by the group are  $\theta_{jt} \int_0^{H^{-1}(\alpha_{jt})} \xi h(\xi) d\xi$ .

**A family problem:** Collecting the results above, and assuming that aggregate shocks are Markov, we may re-express the lifetime plans formulated by individual households as the solution to the recursive problem of an extended family that manages the joint brokerage account of all households and acts to maximize the equally-weighted sum of their utilities. In each period, given the starting distribution of households summarized by  $\{\theta_j, A_j\}$  and the current price level  $P$ , the family selects the fractions of households from each time-since-active group to receive account transfers,  $\alpha_j$ , (and hence the distribution of households over time-since-active groups at the start of next period,  $\theta'_j$ ), the shopping-time bank balance of each active household,  $M_0$ , achieved by transfers from the family brokerage account, as well as the consumption and money savings associated with members of each shopping-time group,  $c_j$  and  $A'_{j+1}$  respectively, to solve the problem in (13) - (19) below. In solving this problem, the family takes as given the current endogenous aggregate state  $K = [\{\bar{\theta}_j, \bar{A}_j\}, P_{-1}y_{-1}, \bar{M}_{-1}]$ , and it assumes the future endogenous state will be determined by a mapping  $F$  that it also takes as given;  $K' = F(K, s)$ . In equilibrium,  $K'$  is consistent with the family's decisions.

$$V(\{\theta_j, A_j\}; K, s) = \max \sum_{j=1}^{\infty} \theta_j [\alpha_j u(c_0) + (1 - \alpha_j) u(c_j)] + \beta \int_{s'} V(\{\theta'_j, A'_j\}; K', s') g(s, s') ds' \quad (13)$$

subject to:

$$\sum_{j=1}^{\infty} \alpha_j \theta_j [M_0 - M_j] + P \sum_{j=1}^{\infty} \theta_j \left[ \int_0^{H^{-1}(\alpha_j)} \xi h(\xi) d\xi \right] \leq \bar{M} - \bar{M}_{-1} + (1 - \lambda) P_{-1}y_{-1} \quad (14)$$

$$M_j = [A_j + \lambda P_{-1}y_{-1}], \text{ for } j > 0 \quad (15)$$

$$M_j \geq Pc_j + A'_{j+1}, \text{ for } j \geq 0 \quad (16)$$

$$A'_{j+1} \geq 0, \text{ for } j \geq 0 \quad (17)$$

$$\sum_{j=1}^{\infty} \alpha_j \theta_j \geq \theta'_1 \quad (18)$$

$$\theta_j (1 - \alpha_j) \geq \theta'_{j+1}, \text{ for } j > 0 \quad (19)$$

Recall from equation 12 that money market clearing in each period requires that the aggregate of households' current bonds less their expenditures on new bonds must equal the change in the aggregate money supply. By imposing this equilibrium condition, we may use equation 14 to represent the family's budget constraint requiring that its joint brokerage assets cover all current transfers to active households and associated fixed costs, as well as all bond purchases for the next period. We obtain equation 14 as follows. From (34) derived in the proof of Lemma 2 (Appendix), we have the following period-by-period aggregate constraints:

$$B(s^t) - \int_{s_{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}) ds_{t+1} + (1 - \lambda) P(s^{t-1}) y(s^{t-1}) \geq \int_{\xi^t} h(\xi^t) [x(s^t, \xi^t) + P(s^t) \xi_t] z(s^t, \xi^t) d\xi^t.$$

From the monetary authority's constraints in (11), we replace the first two terms on the left-hand side  $\overline{M} - \overline{M}_{-1}$ , adapting our notation somewhat to subsume  $s^t$  and  $s^{t-1}$ . On the right-hand side, we use Lemmas 2 - 4 to track households ( $\xi^t$ ) by their membership in time-since-active groups ( $j$ ) in computing the total transfers to active households and associated fixed costs. Because each active household in group  $j$  receives transfer  $x_j = M_0 - M_j$ , and the fraction of the group becoming active is  $\alpha_j \equiv H(\xi_j^T)$ , total transfers to members of group  $j$  are  $\theta_j \alpha_j [M_0 - M_j]$ . Only transfer costs at or below  $\xi_j^T$  are paid by the members of group  $j$ , so the nominal value of total transfer costs paid from the group is  $P \theta_j \int_0^{H^{-1}(\alpha_j)} \xi h(\xi) d\xi$ .

Next, equation 15 identifies the start-of-period money balances associated with each time-since-active group  $j$ , and (16)-(17) represent the bank account budget and cash-in-advance constraints that apply to members of each shopping-time group. Finally, equations 18 - 19 describe the evolution of households across groups over time. In (18), the total active households (shopping in group 0) in the current period is the population-weighted sum of the fractions of households made active from each start-of-period group, and these households move together to begin the next period in time-since-active group 1. In (19), households in any given time-since-active group  $j$  that are inactive in the current period will move into the next period as members of group  $j + 1$ .

## 2.4 Solution

Recall that we imposed money-market clearing in formulating the family's problem above. As such, we can retrieve equilibrium allocations as the solution to (13) - (19) by appending to that problem the goods market clearing condition needed to determine the equilibrium price level taken as given by the family:

$$y = c_0 \sum_{j=1}^{\infty} \theta_j \alpha_j + \sum_{j=1}^{\infty} \theta_j (1 - \alpha_j) c_j + \sum_{j=1}^{\infty} \theta_j \left[ \int_0^{H^{-1}(\alpha_j)} x h(x) dx \right]. \quad (20)$$

Equation 20 simply states that, within each period, the current aggregate endowment must satisfy total consumption demand across all active and inactive households together with the economywide fixed costs associated with account transfers.

In the examples to follow, we abstract from trend growth in endowments, and we assume the money supply grows at the rate  $\mu^*$  in the economy's steady-state. Thus, the steady-state is associated with inflation at rate  $\mu^*$  and a stationary distribution of households over real balances described by  $[\theta^*, \mathbf{a}^*]$ , where  $\theta^* = \{\theta_j^*\}$  and  $\mathbf{a}^* = \{a_j^*\}$ , with  $a_j \equiv \frac{A_j}{P_{-1}}$ . As any given household

travels outward across time-since-active groups, it finds its actual real balances for shopping time,  $(a_j^* + \lambda y^*)/(1 + \mu^*)$ , falling further and further below target shopping balances; thus, the maximum fixed cost it is willing to pay to undertake an account transfer rises. Given a finite upper support on the distribution of fixed transfer costs, this implies that no household will delay activity beyond some finite maximum number of periods, which we denote by  $J$ . Thus, the two vectors describing the distribution of households are each of finite length  $J$ . In solving the steady-state of our economy, we isolate  $J$  as that group  $j$  by which  $\alpha_j$  is chosen to be 1.

Having arrived at the time-since-active representation described above, we can follow King and Thomas (2006) in adopting a sequence notation and applying linear methods to solve for our economy's aggregate dynamics local to the deterministic steady-state. However, in doing so, we must address three remaining details. First, as the linear solution does not allow for a changing number of time-since-active groups, we must restrict  $J$  to be time-invariant. Thus, we assume that  $\alpha_{J,t} = 1$  for all  $t$ , and we then verify that  $\alpha_{j,t} \in (0, 1)$ , for  $j = 1, \dots, J - 1$ , is selected throughout our simulations. Second, we assume that, in every date  $t$ , all households that enter shopping in time-since-active group  $J - 1$  completely exhaust their money balances;  $a_{J,t} = 0$ . Given that any such household will undertake an account transfer with certainty at the start of the next period, this assumption is consistent with optimizing behavior so long as we verify that nominal interest rates are always positive.<sup>11</sup> Third, we assume shocks are never so large that a household will exhaust its cash before it is sure to become active; that is, we assume  $a_{j,t} > 0$  for all groups  $j < J$  at all dates  $t$  and confirm that this is satisfied throughout our simulations.

### 3 Examples

We begin to explore our model's dynamics in this section through a series of examples considering the response to an unanticipated rise in the money growth rate that, once observed, is known to be perfectly transitory. Throughout this section, we abstract from shocks to the endowment to study the effects of a monetary shock in isolation, and to isolate those aspects caused by the endogenous changes in the degree of market segmentation that distinguish our model.

In parameterizing the endowment economy model, we set the length of a period to one quarter, and we choose the steady-state inflation rate  $\mu^*$  to imply an average annual inflation at 3 percent. Period utility is iso-elastic,  $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ , with  $\sigma = 2$ , and we select the subjective discount factor  $\beta$  to imply an average annual real interest rate of 3 percent. The steady-state aggregate endowment is normalized to 1, and the fraction of the endowment paid to household bank accounts (which may be interpreted as household wages) is  $\lambda = 0.6$ , corresponding to labor's share of output.

We consider two cases of our model distinguished only by the distribution of the fixed transfer costs that cause its market segmentation. We begin with a baseline case where this distribution is uniform on the interval 0 to 0.25. There, the maximum time that any household remains inactive is  $J = 6$  quarters, while the average duration between account transfers is 4.82 quarters. In the aggregate, this parameterization results in a steady-state velocity of 1.9, which corresponds to the U.S. average between 1995Q1 and 2005Q1.<sup>12</sup> In our second example, we move to consider a more

<sup>11</sup>Given positive nominal rates, if  $a_{J,t} > 0$  ever occurred, the family could have achieved higher welfare by reducing the balances given to active households at date  $t - (J - 1)$  and increasing bond purchases at that date to finance increased transfers to a subsequent group of active households for whom the non-negativity constraint would bind.

<sup>12</sup>We follow Alvarez, Atkeson and Edmond (2003) in our measures of money and velocity. Money is broadly defined as the sum of currency, checkable deposits, and time and savings deposits. Alvarez et al. show that the opportunity cost of these assets, relative to short-term Treasury securities, is substantial and, as a whole, not very different from

flexible cost distribution where the maximum transfer cost is raised to imply an aggregate velocity matching the U.S. postwar average, at 1.42. This second distribution is sharply right-skewed and implies a mean household inactivity duration of 9.55 quarters that is very close to the maximum inactivity spell, 10 quarters. Its choice is prompted by the micro-level data discussed below.

Before examining its responses to shocks, it is useful to begin with a discussion of household portfolio adjustment timing in our model's steady-state. We first consider how each of our examples relates to the available micro-evidence provided by Vissing-Jørgensen (2002). Using the Consumer Expenditure Survey, Vissing-Jørgensen computes that the fraction of households that actively bought or sold risky assets (stocks, bonds, mutual funds and other such securities that correspond to non-money assets in our model), between one year and the next ranges from 0.29 to 0.53 as a function of financial wealth.<sup>13</sup> For a direct comparison with each version of our quarterly model, we compute the steady-state unconditional probability that a household will undertake active trade within one year as  $\sum_{j=1}^J (\theta_j - \theta_{j+4})$ , with  $\theta_{j+4} = 0$  for  $j > J - 4$ . We find that the fraction of households actively trading in an average year is 0.78 in our baseline example, which is quite high relative to the Vissing-Jørgensen data. This may be explained in part by the fact that the transfer costs in this example are calibrated to match aggregate velocity over the decade ending at 2005, when transactions costs were presumably lower than in her 1982-1996 sample period. When we instead calibrate to match aggregate velocity over the postwar period in our second example with higher transactions costs and a longer mean inactivity episode, the average annual fraction of households conducting trades lies well within the empirical range, at 0.42.<sup>14</sup>

We cannot compare our model-implied mean inactivity durations to the Vissing-Jørgensen data without making some assumption about the shape of the empirical hazard. If one assumes that the probability of an active trade is constant from quarter to quarter in the data, then the Vissing-Jørgensen range reported above implies a mean duration of household inactivity ranging from 7.5 to 13.8 quarters. Recall that the mean duration of inactivity in our baseline example is only 4.8 quarters, whereas our second example with high mean inactivity spells has an average duration of 9.55 quarters, neatly inside the range implied by the data.

We confine our remaining discussion of the model's steady-state to that arising under our baseline parameters, as the qualitative aspects we emphasize hold for both examples. Here, with the aggregate endowment and the money growth rate fixed at their mean values, six groups of households enter into each period, with these groups corresponding to the number of quarters that have elapsed since members' last account transfer. As any given household moves through these groups over time, its real money balances available for shopping fall further and further below the target value, 2.94, given both inflation and its expenditures subsequent to its transfer. To correct this widening distance between actual and target real balances, the household becomes increasingly willing to incur transfer costs. This implies that the threshold cost separating active households from inactive ones rises with households' time-since-active. Thus, as transfer costs are drawn from a common distribution, the fractions of households currently active in Table 1 rise across start-of-period groups; in other words, the model predicts a rising adjustment hazard.

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that of M1. Next, velocity is computed as the ratio of nominal personal consumption expenditures to money.

<sup>13</sup>The CEX interviews about 4500 households each quarter, interviewing each five times and collecting financial information in the final interview. Vissing-Jørgensen (2002) limits her sample to 6770 households that held risky assets both at the time of the fifth interview and one year prior.

<sup>14</sup>In calibrating our production economy below in section 4, we arrive at a cost distribution with similar shape.

**TABLE 1. Determination of steady-state shopping-time distribution**

time-since-active group	0	1	2	3	4	5	6
start-of-period populations		0.208	0.205	0.196	0.174	0.136	0.082
<i>fraction currently active</i>		<i>0.011</i>	<i>0.045</i>	<i>0.113</i>	<i>0.218</i>	<i>0.397</i>	<i>1.000</i>
shopping-time real balances	2.936	2.510	2.095	1.691	1.301	0.929	n/a
shopping-time populations	0.208	0.205	0.196	0.174	0.136	0.082	0

In figure 1A, we plot the steady-state distribution of households across groups as they enter shopping-time from the final row of table 1. Corresponding to the rising hazard shown above, the dashed curve reflecting the measures of households in each shopping-time group monotonically declines across groups. The solid curve in the figure illustrates the ratios of real consumption expenditure relative to real balances, individual velocities, associated with the members of each shopping-time group. Because households are aware that they must use their current balances to finance consumption not only in the current period but also throughout subsequent periods of inactivity, individual spending rates rise across groups in response to a declining expected duration of future inactivity. Currently active households, those households in group 0, face the longest potential time before their next balance transfer, and thus have the lowest individual velocities. By contrast, households currently shopping in group 5 will undertake a transfer with certainty at the start of the next period; thus, individual velocity is 1 for members of this last group.

Two aspects distinguishing our endogenous segmentation model will be relevant in its responses to shocks below. First, on average, a household’s probability of becoming active monotonically rises with the time since its last active date, as seen above. Second, these probabilities change over time as shocks influence the value households place on adjusting their bank balances. To isolate the importance of these two elements below, we will at times contrast the responses in our economy to those in a corresponding economy that has neither. In that otherwise identical *fixed duration* model, the timing of any household’s next account transfer is certain and is not allowed to change with the economy’s state. For comparability with our endogenously segmented economy, where households’ mean duration of inactivity is 4.8 quarters, households in the corresponding fixed duration model undertake transfers exactly once every 5 quarters.

Figure 1B displays the steady-state of the fixed duration model. There, households enter every period evenly distributed across 5 time-since-active groups. Throughout groups 1 through 4, fraction 0 of each group’s members are allowed to undertake account transfers, while fraction 1 of the members of group 5 are automatically made active. Thus, 20 percent of households enter into shopping in each time-since-active group 0 through 4, and this shopping-time distribution remains fixed over time. As in our model with endogenously timed household portfolio adjustments, here too individual velocities monotonically rise with time-since-active and hit 1 in the final shopping group. However, given its lesser maximum duration of inactivity (5 quarters here versus 6 in the endogenous segmentation model), households in the fixed duration economy exhibit somewhat higher spending rates throughout the distribution relative to those in panel A.

### 3.1 Money injection in the baseline case

Throughout the rest of this section, we study the effects of an unanticipated one period rise in the money growth rate using a series of examples designed to illustrate our model’s mechanics. Here, we consider the baseline case corresponding to Table 1, where inactivity durations are short.

*Fixed duration model.* For reference, we begin in figure 2 with an examination of the aggregate response in the fixed duration model, where the fractions of active households across groups are fixed and dictated by  $\alpha_{FD} = [0 \ 0 \ 0 \ 0 \ 1]$ .<sup>15</sup> As seen in the top panel, the aggregate price-level rises only halfway at the date of the money supply shock, with the remaining price adjustment staggered across several subsequent periods. This inflation episode continues until those households who were active at the shock date have traveled through all time-since-active groups and are once again active, at the start of date 6.

The aggregate price-level adjusts gradually in this exogenously segmented markets economy for precisely the reasons explained by Alvarez, Atkeson and Edmond (2009). Given the rate-of-return dominance implied by positive nominal interest rates, open market operations that inject money into the brokerage accounts must be fully absorbed by active households.<sup>16</sup> However, as they will be unable to access their brokerage accounts again for 5 periods, these households retain large inventories of money relative to their current consumption spending. As noted above, their spending rate is the lowest among all households in the economy. Consequently, total nominal spending does not rise in proportion to the money supply, and a rise in the share of money held by active households leads to a rise in aggregate real balances. Equivalently, in this endowment model, velocity falls.

Formally, in a fixed duration model with  $J$  time-since-active groups, aggregate velocity may be expressed as the sum of two terms, one associated with the common velocity of currently active households and one associated with the velocities of each group of inactive households:

$$V_t = \frac{1}{J} \frac{M_{0t}}{\bar{M}_t} v_{0t} + \sum_{j=1}^{J-1} \frac{1}{J} \frac{M_{jt}}{\bar{M}_t} v_{jt}. \quad (21)$$

From this equation, it is clear that the rise in relative money holdings of active households must reduce aggregate velocity, so long as individual velocities do not rise much in response to the shock. As seen in the bottom panel of figure 2, in our fixed duration example, half of the money injection is absorbed by an initial fall in aggregate velocity. As households that were active at the time of the shock travel through time-since-active groups in subsequent periods, their spending rate rises, pulling aggregate velocity back up. During this episode nominal spending rises faster than the money supply, the price level grows above trend, and aggregate real balances return to their long-run level.

Turning to the response in interest rates shown in the middle panel of figure 2, note that the money injection causes a large, but purely transitory, liquidity effect. In economies with segmented markets, real rates are determined by the marginal utilities of consumption among active households in adjacent periods, given that only these households can transform interest-bearing assets into consumption. In the fixed duration model, only those households that are active at the date of the shock experience a rise in their lifetime wealth. As a result, their consumption rises, while the consumption of households active in subsequent dates remains unchanged, thus explaining the large but temporary fall in the real interest rate.

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<sup>15</sup> Figures in this and the subsequent section reflect the effects of a temporary 0.1 percentage point rise in the money growth rate. Given that our model is solved linearly, we have re-scaled all responses to correspond to a 1 percentage point rise for readability.

<sup>16</sup> No household unable to shift assets from its brokerage account into its bank account will accept the additional money, given the rate-of-return dominance implied by positive nominal interest rates.



*Endogenous segmentation model.* Figure 3 displays the aggregate response to the same temporary shock in the baseline case of our model. The endogenous segmentation economy exhibits somewhat sharper initial price adjustment, associated with a smaller fall in aggregate velocity, and it has a more protracted response in inflation. Although the average time between a household's account transfers is 4.8 periods in our model's steady-state, its high inflation episode following the purely transitory money shock lasts 8 periods. The initial decline in interest rates is substantially smaller than were those in figure 2, at about one-tenth of the size of the money growth shock. However, in contrast to the immediate correction seen under fixed duration, the real interest rate here remains persistently low for 6 quarters. These differences in amplitude and propagation arise from the two elements distinguishing our model, the nontrivial rising hazard reflecting the fractions of households undertaking bank transfers from each time-since-active group, and the movement in this hazard in response to an aggregate shock. The first of these elements is central to our model's larger initial rise in inflation, while the second is entirely responsible for its substantially different real interest rate response.

Similar to (21) above, aggregate velocity in our model is determined by a weighted sum of the individual velocities of active and inactive households, with weights determined by the measures of households in each time-since-active group and their relative individual money holdings:

$$V_t = \left( \sum_{j=1}^J \alpha_{j,t} \theta_{j,t} \right) \frac{M_{0,t}}{M_t} v_{0,t} + \sum_{j=1}^{J-1} (1 - \alpha_{j,t}) \theta_{j,t} \frac{M_{j,t}}{M_t} v_{j,t} + \frac{P_t}{M_t} \sum_{j=1}^J \theta_{j,t} \int_0^{H^{-1}(\alpha_j)} x h(x) dx. \quad (22)$$

The final term reflects the proportion of the aggregate money stock used in paying transfer costs, and was absent in (21). However, this term is quantitatively unimportant both on average and following the shock, so it cannot explain our economy's lesser decline in aggregate velocity relative to the fixed duration model. The first-order difference lies in the second term, the weighted velocities of inactive households.

In the fixed duration model, every household spends all of its money between one balance transfer and the next, because this timing is certain. By contrast, the average household in our economy typically has some left-over money in its bank account when it undertakes its next transfer, because this timing is uncertain. Given their ability to alter expected left-over money, our inactive households are more flexible in responding to the money growth shock.<sup>17</sup> In response to the rise in anticipated inflation, their spending rates,  $v_{j,t}$ , rise between 0.3 and 0.5 percent, roughly twice as much as in the fixed duration model. As a result, our economy experiences a lesser decline in the second (and largest) term determining aggregate velocity at the date of the shock due to its nontrivial hazard. This is mitigated to some extent by changes in the hazard, as discussed below.

Because the money injection implies an inflationary episode that will reduce inactive households' real balances, it increases the value of converting bonds held in the brokerage account into money. Thus, a greater than usual measure of households become active. However, the rise in household activity rates has only limited impact in reducing aggregate velocity, since it implies that each active household will receive a lesser share of the total money injection. As a result, the weight  $\frac{M_{0,t}}{M_t}$  is smaller in (22) than it is in (21), which in turn implies a lesser initial rise in the consumption of active households in our economy. The smaller rise in each active household's money holdings also implies that their velocity falls by less than in the fixed duration model (0.4 versus 1.6 percent).

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<sup>17</sup>Inactive households set consumption to satisfy  $u'(c_{j,t}) = \beta E_t \left[ \frac{P_t}{P_{t+1}} (1 - \alpha_{j+1,t+1}) u'(c_{j+1,t+1}) + \frac{P_t}{P_{t+1}} \alpha_{j+1,t+1} u'(c_{0,t+1}) \right]$ . Because they have lower consumption than active households, and  $c_{0,t+1}$  rises with the shock, the probability of becoming active in the next period compounds anticipated inflation in discouraging their money savings.

While endogenous market segmentation reduces the initial real effect of a monetary shock, it also propagates it through changes in the timing of households' transfer activities, which are summarized in panel A of figure 4. Following a substantial initial rise, the overall measure of active households falls below its steady-state value for a number of periods, despite persistently high activity rates across groups,  $\alpha_{jt}$ . This is because large initial rises in these rates shift the household distribution to imply higher than usual money balances for the mean household in subsequent dates, thereby reducing its incentive to transfer funds from the brokerage account. Alternatively, households' initial rises in activity rates thereafter imply unusually high membership in low-numbered time-since-active groups where, given the rising hazard, activity rates are lowest.

Those persistent changes in the distribution of households are responsible for the persistent real effects in our economy. In dates following the shock, although money growth has returned to normal, the measure of active households is sufficiently below average that each such household receives an above-average transfer of real balances in equilibrium. Thus, the rise in the consumption of active households in our economy is not purely transitory as it was in the fixed duration model. Rather, as seen in panel B of figure 4, it returns to steady state gradually as the distribution resettles. This explains why the real interest rate has a much smaller initial decline in our economy, and why it remains persistently low.

Figure 5 verifies the importance of changes in our economy's endogenous timing of household transfer activities by displaying the aggregate response in an otherwise identical model where such changes are not permitted. In this *time-dependent* activity model, a nontrivially rising hazard governs the timing of household account transfers; in fact, it is precisely that from our economy's steady-state in table 1. Here, however, this hazard is held fixed throughout time. From the comparisons in panels A and C, it is clear that changes in group-specific activity rates serve to reduce aggregate velocity in our model economy, yielding more gradual price adjustment, as was argued above. Next, the time-dependent model's interest rate responses in panel B confirm that our economy's persistent liquidity effects in real rates arise entirely from changes in the hazard, rather than its average shape. Absent these changes, the interest rate decline is completely transitory just as in the fixed duration model.

### 3.2 Money injection in the high duration case

In the preceding example, the fixed transfer costs causing our economy's market segmentation were selected to yield average aggregate velocity at 1.9, and implied a 4.8 quarter average duration of inactivity among households. Here, we examine our model's response to the same temporary money growth shock in a setting where transactions costs are drawn from a beta distribution with shape parameters  $[\alpha = 3, \beta = 1/3]$  and an upper support at 0.50. As noted above, this second case yields an average aggregate velocity matching the U.S. postwar average, mean and maximum household inactivity durations at 9.55 and 10 quarters, respectively, and it implies that the fraction of households undertaking active trades within one year is 0.42, inside the range estimated by Vissing-Jørgensen (2002), 0.29 – 0.53.

With this change in the cost distribution, our model's steady-state hazard describing average activity rates is much like that of a fixed duration model, in that activity rates are near zero for all groups below  $J$ . As such, one might imagine that its dynamic response would resemble a  $J = 10$  version of figure 2. However, because our households are able to change the timing of their transfers, this is not the case.

Figure 6 reveals that the response to the temporary money shock is quite similar to that in our

baseline endogenous segmentation example. Again, adjustment in the aggregate price level is slow, resulting in a persistently high inflation episode and, unlike a fixed duration model, the real interest rate is persistently low. The one new feature here relative to both a fixed duration model and our baseline endogenous segmentation example is a persistent liquidity effect in nominal interest rates. From this, it is clear that the distribution of the costs responsible for market segmentation can have important effects on aggregate dynamics.

## 4 Results in an economy with production

The examples we have considered thus far are useful in illustrating the mechanics of our model. However, the impulse responses following a surprise money injection in an endowment economy may not be particularly reflective of inflation and interest rate dynamics in an actual economy. Here, we consider a version of our model with production wherein the monetary authority follows a Taylor rule responding to deviations in inflation and output.

Households in the production economy value both consumption and leisure. Their period utility function is  $u(c, n) = \frac{(c - \eta n^\gamma)^{1-\sigma}}{1-\sigma}$ , where  $c$  is consumption,  $n$  denotes hours of work, and we restrict  $\gamma > 1$  and  $\sigma > 0$ . Because differences in money holdings imply differences in consumption, our introduction of variable labor supply in this section implies differing hours worked for households in different time-since-active groups. We use the notation  $n_{j,t-1}$  to describe the hours of work in period  $t-1$  by a household that, at that time, had last been active  $j$  periods in the past,  $j = 0, \dots, J-1$ .

We assume a perfectly competitive representative firm that hires labor from households at common real wage  $w_t$  and produces the single consumption good. The firm is owned equally by all households, there is no trade in shares, and all profits are returned to owners. Denoting aggregate employment by  $N_t$ , aggregate production is given by:

$$Y_t = z_t N_t^\nu, \quad (23)$$

where  $z_t$  is the current realization of aggregate productivity, and profits are  $\Pi_t = Y_t - w_t N_t$ .

The timing of events within a period is as follows. At the beginning of each period, households observe the current aggregate state, draw their transfer costs, determine their portfolio adjustments, and then participate in the labor and goods markets. Fraction  $\lambda$  of labor and profit income is paid into the bank account. As before, income earned this period is available at the start of the next. Thus, there remains a cash-for-goods constraint on consumption purchases. While we use  $w_{t-1}$  to describe the real wage in period  $t-1$ , and  $\Pi_{t-1}$  to represent real profits, all income is paid in nominal units. Thus, at the beginning of period  $t$ , the real payment into the bank account of each household of time-since-active type  $j$ ,  $j = 1, \dots, J$ , is  $\lambda(w_{t-1}n_{j-1,t-1} + \Pi_{t-1})\frac{P_{t-1}}{P_t}$ , and  $(1-\lambda)(w_{t-1}n_{j-1,t-1} + \Pi_{t-1})\frac{P_{t-1}}{P_t}$  is deposited into the brokerage account.

We represent the current type- $j$  household's real wage earnings from the previous period as  $e_{jt}$  in the problem that follows. The other wealth specific to a household is the money retained in its bank account at the end of the previous period. Let  $a_{jt}$  denote the real value of the money saved by a household currently of type  $j$ , as of the end of last period, which implies real balances of  $\frac{P_{t-1}}{P_t}a_{jt}$  for the current period. A type- $j$  household then has real balances  $m_{jt}$  in its bank account at the start of this period, where:

$$m_{jt} = \frac{P_{t-1}}{P_t}[a_{jt} + \lambda(e_{jt} + \Pi_{t-1})], \text{ for } j = 1, \dots, J. \quad (24)$$

As before, we retrieve competitive allocations by solving the recursive problem of an extended family. Relative to the family objective function (13), the only change is that  $u(c, n)$  replaces the original period utility function  $u(c)$ . Noting that we now use  $a_{j,t}$  to define the support of the distribution of real balances, we briefly describe the constraints involved here. Most are simply real-valued analogues of equations 14 - 19 from section 2.3 with endowment income replaced by labor and profit income. Beyond these, we have the aggregate production function in (23), alongside the new constraints associated with the introduction of variable labor supply. The first set identifies the labor income a household obtains from the current period:

$$w_t n_{jt} \geq e_{j+1,t+1} \text{ for } j = 0, \dots, J - 1. \quad (25)$$

For convenience, we define a new variable  $\chi$  to summarize total income entering the brokerage accounts each period. Specifically, let  $\chi_{t+1}$  represent the end of period  $t$  value of real labor income and profits to be deposited into the family brokerage account at the start of period  $t + 1$ ;

$$(1 - \lambda) \left( \Pi_t + \sum_{j=1}^J \theta_{jt} \alpha_{jt} e_{1,t+1} + \sum_{j=1}^{J-1} \theta_{jt} (1 - \alpha_{jt}) e_{j+1,t+1} \right) \geq \chi_{t+1}. \quad (26)$$

The brokerage account constraint (14) then becomes

$$\frac{P_{t-1}}{P_t} \chi_t + \frac{P_{t-1}}{P_t} \mu_t \bar{a}_{t-1} \geq \sum_{j=1}^J \theta_{jt} \alpha_{jt} (m_{0t} - m_{jt}) + \sum_{j=1}^J \theta_{jt} \varphi(\alpha_{jt}), \quad (27)$$

where  $\varphi(\alpha_{jt}) \equiv \int_0^{H^{-1}(\alpha_j)} \xi h(\xi) d\xi$  as before,  $m_{0t}$  is the shopping-time real balances allocated to each currently active household, and  $\bar{a}_{t-1}$  represents economywide real balances as of the end of date  $t - 1$ .

Turning to the constraints involving households' bank accounts, we replace equations (15) and (16) with their real analogues in (24) and (28).

$$m_{j,t} \geq c_{jt} + a_{j+1,t+1} \text{ for } j = 0, \dots, J \quad (28)$$

For each shopping-time group  $j = 0, \dots, J$ , the cash-for-goods requirement is imposed by  $a_{j+1,t+1} \geq 0$ , the counterpart to (17).

The laws of motion for the distribution of households over time-since-active groups, (18) and (19), continue to apply, and (24) and (28) determine the associated group-specific real balances as a function of lagged real balances and labor incomes,  $\{a_{jt}, e_{jt}\}_{j=1}^J$ , lagged profits,  $\Pi_{t-1}$ , and inflation,  $\pi_t = \frac{P_t}{P_{t-1}} - 1$ . As before, the date- $t$  distribution of real balances is fully described by  $\{\theta_{jt}, m_{jt}\}_{j=1}^J$ .

Given the one-period lag in the deposit of labor and profit income, the family's state vector is now  $(\{\theta_{jt}, a_{jt}, e_{jt}\}_{j=1}^J, \Pi_{t-1}, \chi_t)$ . The aggregate resource constraint in (20) still applies. Equilibrium in the labor market is imposed by:

$$N_t = n_{0t} \sum_{j=1}^J \alpha_{jt} \theta_{jt} + \sum_{j=1}^{J-1} (1 - \alpha_{jt}) \theta_{jt} n_{jt}. \quad (29)$$

Finally, we assume that monetary policy follows a conventional active Taylor rule of the form:

$$i_t = i^* + 1.5[\pi_t - \pi^*] + 0.30[y_t - y^*]. \quad (30)$$

The production economy is parameterized as follows. As before, the length of a period is one quarter, we choose the steady-state inflation rate  $\mu^*$  to imply an average annual inflation at 3 percent, and we set the subjective discount factor  $\beta$  to imply an average annual real interest rate of 3 percent. We set labor income's share to  $\nu = 0.64$  and the fraction of income paid into the bank accounts at  $\lambda = 0.6$ . Next, in specifying period utility, we set  $\sigma = 2$  and  $\gamma = 2.5$ ; this implies a labor supply elasticity intermediate between the range of values conventionally used in macroeconomic research and those from microeconomic studies. We choose  $\eta$  so that aggregate hours worked average  $\frac{1}{3}$  of time, and we normalize mean aggregate productivity so that steady state output is 1.

As in section 3.2, we select a distribution of transactions costs that yields an average aggregate velocity in our model matching the U.S. postwar average, while implying sufficiently infrequent portfolio adjustments among households as to be consistent with the Vissing-Jørgensen (2002) evidence. As was true in the endowment economy setting, this requires a right-skewed cost distribution implying a relatively long mean duration between trades. Here, transactions costs are drawn from a beta distribution with shape parameters  $[\alpha = 4, \beta = 1/4]$  and an upper support at 0.21. The mean time between trades is 7.9 quarters, while the maximum is 8 quarters, and the fraction of households undertaking trades within one year is 0.51.

#### 4.1 Interest rates and aggregate consumption growth

We noted in section 1 that New Keynesian models, and indeed all models assuming a representative household, predict a tight link between real interest rates and the growth rate of aggregate consumption, through the household consumption Euler equation. It is well understood that Euler-implied real interest rates obtained using aggregate consumption data are a poor proxy for actual market rates. However, Canzoneri, Cumby and Diba (2007) show that the discrepancy extends beyond a problem of mean spreads, or even volatilities. Using U.S. consumption data to retrieve the real rate series predicted by the Euler equation under a series of preference specifications both without and with habit persistence of varying forms, they show that the Euler-implied interest rate series is consistently negatively correlated with observed real rates.

An important contribution of our model is its unique consistency with the empirical findings of Canzoneri et al. Here, we establish that, as in the U.S. data, our model's actual real interest rate series is negatively correlated with the series that would be imputed from its aggregate consumption growth rates under the assumption of a representative household. For simplicity, we implement the baseline exercise with constant relative risk aversion preferences, which is as follows. Let  $r^E$  represent the consumption Euler-based real interest rate, let  $C$  represent aggregate consumption, and let  $r^M$  be the market real rate. Given CRRA utility, the representative household's Euler equation is

$$C_t^{-\sigma} = \frac{1}{1 + \rho} E_t[(1 + r_{t+1}^e) C_{t+1}^{-\sigma}],$$

where  $\rho$  is the household's subjective discount rate. Taking logarithms, and imposing certainty equivalence, we infer an approximate ex-post real interest series via

$$r_{t+1}^E \simeq \sigma \gamma_{t+1}^C + \rho, \tag{31}$$

where  $\gamma^C$  is the gross growth rate of aggregate consumption between dates  $t$  and  $t + 1$ .<sup>18</sup> From

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<sup>18</sup>Canzoneri et al. (2007) report that comparing ex-ante rates obtained by adjusting nominal rates with one-quarter ahead inflation forecasts yields the same basic results as the ex-post rate comparison undertaken here.

the equation above and quarterly U.S. consumption data between 1966 and 2004, Canzoneri et al. obtain the Euler-implied interest rate series  $\{r_t^E\}$ . Comparing this to the U.S. Federal Funds rate over the same period  $\{r_t^M\}$ , they find the correlation between the two series is  $-0.37$ .

We conduct this same investigation using data from a 10,000 period simulation of our production economy model. Here, we include aggregate shocks to productivity,

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_t^z,$$

and to the interest rate rule specified in equation 30, now appended by innovations  $\varepsilon_t^i$ . Each shock has mean-zero normal innovations;  $\varepsilon_t^z \sim N(0, \sigma_{\varepsilon^z}^2)$  and  $\varepsilon_t^i \sim N(0, \sigma_{\varepsilon^i}^2)$ . To ensure that our model-based comparison is proximate to the empirical comparison described above, we select the volatilities of the two shocks so that our simulated aggregate consumption growth rates and market real interest rates have standard deviations closely matching those in the data, and we set  $\rho_z$  so that our model reproduces the persistence of U.S. consumption growth rates.<sup>19</sup>

Figure 7 presents a 75-period subsample of interest rate data drawn from our full-length simulation. Based on a quick inspection of the figure, it is easy to see that the Euler-derived interest rate implied by aggregate consumption growth in our model does not co-move positively with the actual real interest rate, which in our environment is determined by the marginal utilities of active (trading) households in adjacent dates. In fact, when we examine the results from the full 10,000 period simulation summarized in Table 2, we see that the model predicts a negative correlation between the two series surprisingly close to that observed in the U.S. data.

**TABLE 2. Consumption growth and real interest rates**

	cons. gr. autocorr.	sd(cons. gr.)	sd( $r^M$ )	corr( $r^M, r^E$ )
U.S. data	0.26	0.70	2.39*	-0.37*
model-generated data	0.26	0.70	2.38	-0.26
fixed segmentation model	-0.01	0.79	5.05	0.23

\*From Table 1 of Canzoneri et al (2007).

As in any model with segmented asset markets, interest rates in our model are not determined by the consumption path of a representative household but, instead, by subsets of households actively trading in the market for government bonds. This fact alone allows some separation between changes in aggregate consumption and the equilibrium real interest rate. However, the final row of Table 2 establishes that the mere presence of asset market segmentation is not sufficient to obtain our model's success. When we consider an otherwise identical version of the model with fixed segmentation, the correlation in the final column is far closer to the data than would be obtained in a representative household model, but it remains positive.<sup>20</sup> To obtain the negative correlation between the market clearing real interest rate and the interest rate implied by aggregate

<sup>19</sup>Market real rate volatility is matched to the standard deviation of the U.S. Federal Funds rate reported by Canzoneri et al. (2007). Aggregate consumption growth rate volatility is matched to the standard deviation of quarterly real PCE growth rates in the U.S. over 1954:Q1 through 2005:Q1. The resulting parameter values are  $\sigma_{\varepsilon^i}^2 = 0.0024$ ,  $\sigma_{\varepsilon^z} = 0.0033$  and  $\rho_z = 0.975$ .

<sup>20</sup>The fixed segmentation model is the time-dependent model wherein the hazard governing household activity rates is fixed at the steady state hazard implied by our model. Given the sharply right-skewed cost distribution in our model, this time-dependent model closely resembles a fixed duration model with Taylor adjustment timing.

consumption growth, we must permit the extent of market segmentation to change over time in response to aggregate conditions.

Across both models in Table 2, the correlation between Euler-derived and market real interest rates is largely determined by the model's responses to productivity shocks. To explore why our model succeeds in generating a negative correlation while the fixed segmentation model does not, we consider the mechanics of each following a persistent shock to aggregate total factor productivity. Figure 8 presents responses in the series most relevant to this discussion.

Absent any policy response, the rise in aggregate output following a persistent rise in aggregate productivity implies a fall in inflation. Thus, in each model economy, the Taylor rule dictates that the monetary authority must respond by lowering the nominal interest rate. Given that the interest rate rule is active, the nominal rate is reduced by more than the reduction in inflation. As a result, the market real interest rate falls.

In our model, endogenous changes in the extent of asset market segmentation introduce persistence in the responses in inflation and interest rates. The reasons for this are reminiscent of the effects of time-variation in household activity rates in the examples explored in section 3. As before, the initial fall in the nominal interest rate induces a rise in the number of active households. This reduces the rise in consumption achieved by each active household at the date of the shock, and it permits a rise in the consumption of households active over subsequent dates. As a result, we see a persistent fall in the market real interest rate in the bottom panel of Figure 8.

Aggregate consumption in our model exhibits a hump-shaped response unusual in settings without capital investment, as shown in the top panel of Figure 8. This happens because changes in activity rates spread the benefits of the productivity shock across households active in dates beyond the impact of the shock. Because households active in subsequent dates also carry real balances exceeding the level typically held by an active household, they too increase their consumption spending, driving further rises in production. As a result, aggregate consumption growth is positive, though falling, for several periods. Given the relation between aggregate consumption growth and the Euler-implied real interest rate in (31), this episode of low real interest rates and high consumption growth rates drives a pronounced negative correlation between the Euler-implied real rate and the market real interest rate.

By contrast, the fixed segmentation model exhibits a larger initial fall in inflation when productivity rises. There, households active in the bond markets at the date of the shock experience a rise in their real balances that finances the subsequent purchases of the economy's increased output. As in section 3, without endogenous changes in participation rates, these benefits go exclusively to the initial group of active households. Those active in subsequent periods experience little change in their consumption. As a result, the market real interest rate falls sharply at the date of the shock, but lacks persistence. Moreover, there is a familiar monotone response in aggregate consumption typically seen in models without capital. As aggregate consumption returns to its long run level, consumption growth is slightly negative. Together, the sharp initial fall in the real interest rate and the persistent period of below-average aggregate consumption growth rates yield a weak positive correlation between Euler-implied and market real interest rates.

Our model's success in delivering a negative correlation between the market real interest rate and the Euler-implied rate comes from the persistence in inflation, interest rates and consumption that endogenous asset market segmentation introduces. When households are able to change the timing of their participation in assets markets in response to an aggregate shock, the consequences of the shock are shared far more evenly across households and, as a result, spread over time. This

is essential to the hump-shaped response in aggregate consumption, and thus the episode of positive consumption growth, concurrent with low market real interest rates

## 4.2 Persistence

We showed in section 3 that endogenous changes in household activity can add persistence to inflation and interest rates in an endowment economy setting. In our final section of results, we confirm that this second important contribution of our model survives the extension to a setting with endogenous production and a realistic monetary policy rule. Table 3 compares the second moments of our full model to those of the model with time-invariant segmentation.

**TABLE 3. Further implications of endogenizing segmentation**

	real rate	inflation rate	nominal rate	employment*
<i>% standard deviations</i>				
Our Model	2.38	7.93	8.63	0.61
Fixed Segmentation	5.05	8.29	9.36	0.68
<i>AR(1) coefficients</i>				
Our Model	0.24	0.91	0.87	0.17
Fixed Segmentation	-0.30	0.78	0.73	-0.01
<i>AR(2) coefficients</i>				
Our Model	-0.05	0.85	0.73	0.01
Fixed Segmentation	-0.05	0.77	0.55	-0.01

\*Final column is employment growth rates; remaining columns are level variables reported as annual.

In the discussion surrounding our endowment economy examples, we noted that there is a much smaller rise in inequality following an monetary shock if households are allowed to adjust the timing of their asset market participation. We found that, by spreading the changes in active households' real balances across not only households active at the impact at the shock but also subsequent groups of active households, these changes in participation rates led to more moderate and persistent changes in inflation, and in interest rates. The same observations apply when we confront our production model with a productivity shock.<sup>21</sup>

Table 3 shows that real interest rate volatility is far more moderate in our model than under time-invariant segmentation, while inflation volatility is somewhat muted.<sup>22</sup> The subsequent rows of the table confirm that our model delivers persistence in the real interest rate, where we would otherwise have negative serial correlation. Again, this follows from the fact that shocks generate lesser rises in inequality when participation rates are state-dependent. Time-varying participation rates also yield more persistent movements in inflation by prolonging changes in aggregate velocity. Given the properties of the series in these first two columns, we also see greater persistence in the nominal interest rate.

Throughout this study, we have confined our attention to the behavior of interest rates and inflation. In closing these results, we note that the implications of endogenizing segmentation also extend to quantity variables. As suggested by the consumption moments reported in Table 2, real economic activity evolves more gradually in a setting where the extent of asset market participation is permitted to change in response to aggregate conditions. This is confirmed by the final column

<sup>21</sup>This figure is available on request.

<sup>22</sup>While real and nominal interest rate changes are dampened by changes in activity rates following a shock to the Taylor rule, inflation rate movements are amplified.



of Table 3. Employment growth rates are less volatile when segmentation is endogenous, and they exhibit positive serial correlation where there would otherwise be none.

We have noted above that our endogenous timing of household portfolio adjustments delivers unusual persistence in aggregate consumption for a setting without capital investment. While sufficient to reproduce the empirical first-order autocorrelation of consumption growth, Table 4 reveals that this mechanism alone is insufficient to deliver higher order autocorrelations in growth rates observed in the data.

**TABLE 4. Autocorrelation in aggregate consumption growth**

	first-order	second-order	third-order
U.S. data	0.26	0.21	0.19
model-generated data	0.26	0.01	-0.06
fixed segmentation model	-0.01	-0.01	-0.00

It is worth emphasizing, however, that our model’s success in the first column of Table 4 is evidence of its success with respect to the first two lags in the aggregate consumption series itself. This is all that was needed to explain the interest rate puzzle we set out to explore.

Ours is arguably the first step toward a full-fledged quantitative business cycle evaluation of a new class of monetary models built on endogenous asset market segmentation. However, absent investment, aggregate consumption and output are essentially the same in our current model, apart from small movements in the transactions costs paid by households. Thus, it delivers tight comovement between the growth rates of consumption and hours worked, whereas the correlation between these series in the data is relatively weak (0.289). Similarly, the model does not reproduce the higher variability of employment growth compared to consumption growth, and our resulting standard deviation in employment growth (0.61) is roughly half that in the data (1.21).

## 5 Concluding remarks

In the sections above, we have developed a monetary model where fixed transactions costs lead households to reallocate their wealth between money and interest-bearing securities infrequently. Given these costs, some fraction of households choose not to adjust their portfolios in any given period. Thus, on average, households carry inventories of money to finance their consumption spending over multiple periods, and the price level responds gradually following aggregate shocks. As in any segmented asset markets environment, open market operations directly affect only a subset of all households. Our model is unique in that the size and composition of that subset varies over time as households respond to changes in economic conditions by varying the timing of their portfolio adjustments.

Our approach to endogenizing market segmentation has emphasized idiosyncratic risk at the household level, through the assumption that transactions costs vary randomly across households. We have seen through a series of examples that the underlying distribution of these costs can have important implications for the resulting distribution of money holdings across households, and thus for the economy’s aggregate responses to shocks. Given that fact, we have restricted the distribution of transactions costs to ensure consistency with both macroeconomic data on aggregate velocity, and microeconomic data on the frequency with which households buy or sell securities.

Arguably, an essential criterion for any model attempting to explain how monetary policy affects the aggregate economy is that it should be consistent with the relation between short-term interest rates and real economic activity. We have shown that our model offers two important contributions towards achieving that consistency. First, it effortlessly delivers persistent liquidity effects. Second, it resolves a prevalent difficulty facing models of monetary economies, their counterfactual prediction of positive co-movement between aggregate consumption growth and short-term real interest rates. These results distinguish our environment relative to both sticky price models assuming a representative household and exogenous segmentation models imposing fixed rules for the timing of households' portfolio adjustments.

While we have developed a rich, heterogeneous agent economy that remains tractable, in the spirit of Alvarez et al. (2002), additional elements may be necessary to reconcile our model with some aspects of microeconomic evidence on the frequency with which households trade between risky, relatively illiquid assets and the assets summarized as money here. For example, Vissing-Jørgensen (2002) finds that the frequency of such trades, while on average consistent with that in our quantitative analysis, rises in household financial wealth. Elsewhere, Alvarez et al. (2012) find individuals reporting frequencies of financial asset trades only weakly correlated with their measure of liquidity. Extensions of our model to include permanent differences across households would allow us to address such data with little change in our method and would be unlikely to alter the aggregate results we have emphasized here.

It is straightforward to generalize our model to permit differences in initial wealth (assigning each household to one of  $N$  wealth groups), which could reconcile it with the Vissing-Jørgensen evidence. Alternatively, by assuming that different types of households draw from differing distributions of fixed costs, with these types determined once and for all in the initial period, our analysis and results should continue to apply while reconciling our environment with the Alvarez et al. (2013) evidence. Such extension would imply  $N$  distinct portfolio adjustment hazards and thus model-generated data that, when not controlled for type, could suggest a weak or even positive relationship between liquidity and the probability of portfolio adjustment (see the distinction between true hazards and measured hazards stressed by Cooper et al. (1999)). We can generalize our model for heterogeneity in other respects so long as the assumption of permanent types is maintained, for instance allowing differences in claims on the aggregate endowment (or, in the production economy, differences in skills), or differing fractions of income paid to bank accounts.<sup>23</sup> Extensions to accommodate the addition of persistent heterogeneity (for example, type-specific transfer cost distributions with random changes in types) would be much more involved and require a solution method similar to that developed in King and Thomas (2006), as implemented by Dotsey et al. (2013).

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<sup>23</sup>We show how these extensions and those mentioned in the text above can be implemented in an appendix available on request.

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# Appendix

## A Risk sharing

As described in the main text, bonds issued by a competitive financial intermediary are contingent on both aggregate and idiosyncratic shocks. The supply of these bonds is created using purchases of government bonds and free entry into intermediation implies a zero-profit condition.

Our first lemma derives the equilibrium price of households' bonds. Given the aggregate history  $s^t$ , the intermediary's profit when next period's aggregate shock is  $s_{t+1}$  solves the problem in (8) - (9). There, recall that households identified by the history  $\xi^t$  purchase  $B(s^t, s_{t+1}, \xi^t, \xi_{t+1})$  units of nominal bonds that pay one unit of currency next period if the aggregate shock is given by  $s_{t+1}$  and their idiosyncratic shock is  $\xi_{t+1}$ , and the measure of such households is  $h(\xi^t)$ .

**Lemma 1** *The equilibrium price of state-contingent bonds issued by the financial intermediary,  $q(s^t, s_{t+1}, \xi_{t+1})$ , is given by  $q(s^t, s_{t+1}, \xi_{t+1}) = q(s^t, s_{t+1}) h(\xi_{t+1})$ .*

**Proof.** *Substitution of the right-hand side of (9) for  $B(s^t, s_{t+1})$  in (8) gives*

$$\begin{aligned}
 & \int_{\xi^t} \int_{\xi_{t+1}} q(s^t, s_{t+1}, \xi_{t+1}) B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi^t) d\xi_{t+1} d\xi^t \\
 & - q(s^t, s_{t+1}) \int_{\xi^t} \int_{\xi_{t+1}} B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi^t) h(\xi_{t+1}) d\xi_{t+1} d\xi^t \\
 = & \int_{\xi^t} \int_{\xi_{t+1}} \left( q(s^t, s_{t+1}, \xi_{t+1}) - q(s^t, s_{t+1}) h(\xi_{t+1}) \right) h(\xi^t) B(s^t, s_{t+1}, \xi^t, \xi_{t+1}) d\xi_{t+1} d\xi^t \\
 = & 0.
 \end{aligned}$$

*The last equality follows from the zero profit condition. ■*

## B Characterizing household behavior under risk-sharing

This section provides the main results we use in our numerical approach to solving the model. Lemma 2 proves that, given risk-sharing in the brokerage account, beginning of period money balances capture all relevant differences across households at the start of any period. Additional results in lemma 3 establish that whenever households access their brokerage account, there is uniformity of actions in that they choose the same consumption and bank account balance regardless of their past history of idiosyncratic shocks. The importance of this is that any differences across households will be limited by the number of periods since active transactions between the bank and brokerage account. Such differences disappear whenever households with different bank account balances access their brokerage account at the same time. Finally, lemma 4 proves that households' follow a threshold rule in determining the timing of their account transfers; specifically, it shows that households choose to become active whenever their current fixed transfer cost falls below some threshold value that is common to all households with the same beginning-of-period bank balance.

We begin by deriving a lifetime budget constraint associated with the brokerage account. Using lemma 1, the brokerage budget constraint for any household in the initial period is:

$$\bar{B} \geq \int_{s_1} \int_{\xi_1} q(s_1) h(\xi_1) B(s_1, \xi_1) d\xi_1 ds_1. \tag{32}$$

In period 1, the brokerage account constraint is:

$$B(s_1, \xi_1) \geq \int_{s_2} \int_{\xi_2} q(s_1, s_2) h(\xi_2) B(s_1, s_2, \xi_1, \xi_2) d\xi_2 ds_2 \\ - (1 - \lambda) P(s_0) y(s_0) + \left( x(s_1, \xi_1) + P(s_1) \xi_1 \right) z(s_1, \xi_1)$$

which, substituted into (32), gives:

$$\bar{B} \geq \int_{s_1} \int_{\xi_1} q(s_1) h(\xi_1) \left[ \int_{s_2} \int_{\xi_2} q(s_1, s_2) h(\xi_2) B(s_1, s_2, \xi_1, \xi_2) d\xi_2 ds_2 - (1 - \lambda) P(s_0) y(s_0) \right. \\ \left. + \left( x(s_1, \xi_1) + P(s_1) \xi_1 \right) z(s_1, \xi_1) \right] d\xi_1 ds_1 \\ = \int_{s^2} \int_{\xi^2} q(s^2) h(\xi^2) B(s_1, s_2, \xi_1, \xi_2) d\xi^2 ds^2 - \int_{s_1} \int_{\xi_1} q(s_1) h(\xi_1) \left[ (1 - \lambda) P(s_0) y(s_0) \right. \\ \left. - \left( x(s_1, \xi_1) + P(s_1) \xi_1 \right) z(s_1, \xi_1) \right] d\xi_1 ds_1,$$

where we use the notation  $q(s^t) = q(s_1) \cdot q(s_1, s_2) \cdots q(s^{t-1}, s_t)$  and  $h(\xi^t) = h(\xi_1) \cdots h(\xi_t)$ . Repeated substitutions for  $B(s^j, \xi^j)$  for  $j = 2, \dots, t$  using (3) leads to the following equation.

$$\bar{B} \geq \int_{s^t} \int_{\xi^t} q(s^t) h(\xi^t) B(s^t, \xi^t) d\xi^t ds^t - \sum_{j=1}^{t-1} \int_{s^j} \int_{\xi^j} q(s^j) h(\xi^j) \left[ (1 - \lambda) P(s^{j-1}) y(s^{j-1}) \right. \\ \left. - \left( x(s^j, \xi^j) + P(s^j) \xi_j \right) z(s^j, \xi^j) \right] d\xi^j ds^j.$$

Taking the limit of the above equation, given the No-Ponzi condition,

$$\lim_{t \rightarrow \infty} \int_{s^t} \int_{\xi^t} q(s^t) h(\xi^t) B(s^t, \xi^t) ds^t d\xi^t \geq 0,$$

we arrive at the following lifetime budget constraint associated with the brokerage account.

$$\bar{B} \geq \sum_{t=1}^{\infty} \int \int q(s^t) h(\xi^t) \left( z(s^t, \xi^t) \left[ x(s^t, \xi^t) + P(s^t) \xi_t \right] - P(s^{t-1}) (1 - \lambda) y(s^{t-1}) \right) d\xi^t ds^t \quad (33)$$

Equation 33, which is an immediate implication of the availability of a complete set of state-contingent bonds for each household, implies that individual histories are irrelevant beyond current money balances in the bank account. This intuitive property of the model is straightforward, if notationally cumbersome, to prove and is important in our approach to characterizing competitive equilibrium.

**Lemma 2** *Given  $M(s^{t-1}, \xi^{t-1})$ , the decisions  $c(s^t, \xi^t)$ ,  $A(s^t, \xi^t)$ ,  $x(s^t, \xi^t)$  and  $z(s^t, \xi^t)$  are independent of the history  $\xi^{t-1}$ .*

**Proof.** Equation 33 is equivalent to the following sequence of period by period constraints,

$$B(s^t) \geq \int_{s^{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}) ds_{t+1} - (1 - \lambda) P(s^{t-1}) y(s^{t-1}) \quad (34) \\ + \int_{\xi^t} h(\xi^t) \left[ x(s^t, \xi^t) + P(s^t) \xi_t \right] z(s^t, \xi^t) d\xi^t,$$

and the limit condition

$$\lim_{t \rightarrow \infty} \int_{s^t} q(s^t) B(s^t) ds^t = 0. \quad (35)$$

Let  $\xi_t^{t+j} = \{\xi_t, \dots, \xi_{t+j}\}$  be a partial history of individual household shocks, fix  $(s^t, \xi^t)$  and define pointwise the continuation value to the household's problem given  $(B(s^t), M(s^{t-1}, \xi^{t-1}))$  as

$$V(B_0, M_0; s^t, \xi^t) = \max \sum_{j=0}^{\infty} \beta^j \int_{s_t^{t+j}} \int_{\xi_t^{t+j}} u(c[s^{t+j}, \xi_t^{t+j}]) g(s^{t+j} | s^t) h(\xi_t^{t+j}) ds_t^{t+j} d\xi_t^{t+j} \quad (36)$$

subject to:

$$\begin{aligned} B(s^{t+j}) &= \int_{s_{t+j+1}} q(s^{t+j}, s_{t+j+1}) B(s^{t+j}, s_{t+j+1}) ds_{t+j+1} - (1-\lambda) P(s^{t+j-1}) y(s^{t+j-1}) \\ &+ \int_{\xi_t^{t+j}} h(\xi_t^{t+j}) \left[ x(s^{t+j}, \xi_t^{t+j}) + P(s^{t+j}) \xi_{t+j} \right] z(s^{t+j}, \xi_t^{t+j}) d\xi_t^{t+j}, \quad \text{for } j = 0, \dots, \end{aligned} \quad (37)$$

$$P(s^{t+j}) c(s^{t+j}, \xi_t^{t+j}) \leq M(s^{t+j-1}, \xi_{t-1}^{t+j-1}) + x(s^{t+j}, \xi_t^{t+j}) z(s^{t+j}, \xi_t^{t+j}) - A(s^{t+j}, \xi_t^{t+j}), \quad \text{for } j = 0, \dots, \quad (38)$$

$$M(s^{t+j}, \xi_t^{t+j}) \leq A(s^{t+j}, \xi_t^{t+j}) + \lambda P(s^{t+j}) y(s^{t+j}) \quad \text{and} \quad A(s^{t+j}, \xi_t^{t+j}) \geq 0, \quad \text{for } j = 0, \dots, \quad (39)$$

$$\text{given } B(s^t) = B_0, \quad M(s^{t-1}, \xi_{t-1}^{t-1}) = M_0 \quad \text{and} \quad \lim_{j \rightarrow \infty} \int_{s_t^{t+j}} q(s^{t+j}) B(s^{t+j}) ds_t^{t+j} = 0.$$

Clearly the optimal choices of  $c(s^{t+k}, \xi_t^{t+k})$ ,  $A(s^{t+k}, \xi_t^{t+k})$ ,  $x(s^{t+k}, \xi_t^{t+k})$  and  $z(s^{t+k}, \xi_t^{t+k})$ , for any  $k = 0, 1, \dots$ , are independent of  $\xi^{t-1}$  given  $M_0$ . Let  $c(s^{t+k}, \xi_t^{t+k}; M_0)$ ,  $A(s^{t+k}, \xi_t^{t+k}; M_0)$ ,  $x(s^{t+k}, \xi_t^{t+k}; M_0)$  and  $z(s^{t+k}, \xi_t^{t+k}; M_0)$  describe these choices. Next, any solution to the household's problem must also solve the following problem.

$$\begin{aligned} \max \quad & \sum_{j=1}^{t-1} \beta^{j-1} \int_{s^j} \int_{\xi^j} u(c(s^j, \xi^j)) g(s^j) h(\xi^j) ds^j d\xi^j \\ & + \beta^t \int_{s^t} \int_{\xi^t} V(B(s^t), M(s^{t-1}, \xi^{t-1}); s^t, \xi^t) g(s^t) h(\xi^t) ds^t d\xi^t \end{aligned}$$

subject to

$$\begin{aligned} B(s^j) &= \int_{s_{j+1}} q(s^j, s_{j+1}) B(s^j, s_{j+1}) ds_{j+1} - (1-\lambda) P(s^{j-1}) y(s^{j-1}) \\ &+ \int_{\xi^j} h(\xi^j) \left[ x(s^j, \xi^j) + P(s^j) \xi_j \right] z(s^j, \xi^j) d\xi^j, \end{aligned}$$

$$P(s^j) c(s^j, \xi^j) \leq M(s^{j-1}, \xi^{j-1}) + x(s^j, \xi^j) z(s^j, \xi^j) - A(s^j, \xi^j),$$

$$M(s^j, \xi^j) \leq A(s^j, \xi^j) + \lambda P(s^j) y(s^j) \quad \text{and} \quad A(s^j, \xi^j) \geq 0, \quad \text{for } j = 1, \dots, t-1.$$

In this problem, the optimal choices of  $c(s^{t+k}, \xi_t^{t+k})$ ,  $A(s^{t+k}, \xi_t^{t+k})$ ,  $x(s^{t+k}, \xi_t^{t+k})$  and  $z(s^{t+k}, \xi_t^{t+k})$ , for any  $k = 0, 1, \dots$ , are given by the functions  $c(s^{t+k}, \xi_t^{t+k}; M(s^{t-1}, \xi^{t-1}))$ ,

$A(s^{t+k}, \xi_t^{t+k}; M(s^{t-1}, \xi^{t-1}))$ ,  $x(s^{t+k}, \xi_t^{t+k}; M(s^{t-1}, \xi^{t-1}))$  and  $z(s^{t+k}, \xi_t^{t+k}; M(s^{t-1}, \xi^{t-1}))$ , which attained the maximum value  $V(B(s^t), M(s^{t-1}, \xi^{t-1}); s^t, \xi^t)$ . ■

We proceed to further characterize household behavior by studying the state-contingent plans chosen by households in period 0 when they are all identical. Let  $\Lambda$  denote the multiplier associated with equation 33 and  $\nu_0(s^t, \xi^t)$  be the LaGrange multiplier for (4) and  $\nu_1(s^t, \xi^t)$  the multiplier for (5). For clarity, the Lagrangian is shown below.

$$\begin{aligned} \mathcal{L} = & \sum_{t=1}^{\infty} \beta^{t-1} \int_{s^t} \int_{\xi^t} \left( u(c(s^t, \xi^t)) g(s^t) h(\xi^t) \right. \\ & + \nu_0(s^t, \xi^t) g(s^t) h(\xi^t) \left[ M(s^{t-1}, \xi^{t-1}) + x(s^t, \xi^t) z(s^t, \xi^t) - A(s^t, \xi^t) - P(s^t) c(s^t, \xi^t) \right] \\ & + \nu_1(s^t, \xi^t) g(s^t) h(\xi^t) \left[ A(s^t, \xi^t) + \lambda P(s^t) y(s^t) - M(s^t, \xi^t) \right] \Big) d\xi^t ds^t \\ & + \Lambda \left( \bar{B} + \sum_{t=1}^{\infty} \int_{s^t} q(s^t) (1 - \lambda) P(s^{t-1}) y(s^{t-1}) ds^t \right. \\ & \left. - \sum_{t=1}^{\infty} \int_{s^t} \int_{\xi^t} q(s^t) h(\xi^t) \left( x(s^t, \xi^t) + P(s^t) \xi_t \right) z(s^t, \xi^t) d\xi^t ds^t \right) \end{aligned}$$

Given any choice of  $z(s^t, \xi^t)$ , the household's choices of  $c(s^t, \xi^t)$ ,  $A(s^t, \xi^t)$ ,  $M(s^t, \xi^t)$  and  $x(s^t, \xi^t)$ , satisfy the following conditions.

$$Du(c(s^t, \xi^t)) - P(s^t) \nu_0(s^t, \xi^t) = 0 \quad (40)$$

$$\begin{aligned} -\nu_0(s^t, \xi^t) + \nu_1(s^t, \xi^t) &\leq 0, \quad (41) \\ &= 0 \text{ if } A(s^t, \xi^t) > 0, \end{aligned}$$

$$\begin{aligned} -\nu_1(s^t, \xi^t) + \beta \int_{s_{t+1}} \int_{\xi_{t+1}} \nu_0(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi_{t+1}) g(s_{t+1} | s^t) ds_{t+1} d\xi_{t+1} &\leq 0 \quad (42) \\ &= 0 \text{ if } M(s^t, \xi^t) > \lambda P(s^t) y(s^t) \end{aligned}$$

$$\beta^{t-1} \nu_0(s^t, \xi^t) g(s^t) h(\xi^t) - \Lambda q(s^t) h(\xi^t) = 0 \text{ if } z(s^t, \xi^t) = 1. \quad (43)$$

The following lemma shows that households choose the same consumption and subsequent money balances for their bank account whenever they pay transfer costs to access their brokerage accounts. Recall that this result is important for the approach we take to characterizing household behavior in that it implies that heterogeneity across households is limited to periods when they are inactive.

**Lemma 3** *For any  $(s^t, \xi^t)$  in which  $z(s^t, \xi^t) = 1$ ,  $c(s^t, \xi^t)$ ,  $A(s^t, \xi^t)$  and  $M(s^t, \xi^t)$  are independent of  $\xi^t$ .*

**Proof.** Given  $z(s^t, \xi^t) = 1$ , (40) and (43) imply

$$Du(c(s^t, \xi^t)) = \frac{\Lambda q(s^t) P(s^t)}{\beta^{t-1} g(s^t)}. \quad (44)$$

As  $\Lambda$  is the same for all households, this proves that  $c(s^t, \xi^t)$  is independent of  $\xi^t$ . Next, using lemma 2, notice that strict monotonicity of  $u(c(s^t, \xi^t))$ , for any  $(s^t, \xi^t)$ , implies (38) and (39) will



always bind, hence  $V(B(s^{t+1}), M(s^t, \xi^t); s^t, \xi^t)$  is strictly increasing in  $M(s^t, \xi^t)$ . Without loss of generality, we consider the case where  $A(s^t, \xi^t) > 0$  and therefore  $M(s^t, \xi^t) > \lambda P(s^t) y(s^t)$ . In this case, (41) and (42) together give

$$\nu_0(s^t, \xi^t) = \beta \int_{s_{t+1}} \int_{\xi_{t+1}} \nu_0(s^t, s_{t+1}, \xi^t, \xi_{t+1}) h(\xi_{t+1}) g(s_{t+1} | s^t) ds_{t+1} d\xi_{t+1},$$

which, using (40) and (44) leads to the expression,

$$Du(c(s^t, \xi^t)) = \beta \int_{s_{t+1}} \int_{\xi_{t+1}} Du(c(s^{t+1}, \xi^{t+1})) \left[ \frac{P(s^{t+1})}{P(s^t)} \right]^{-1} h(\xi_{t+1}) g(s_{t+1} | s^t) ds_{t+1} d\xi_{t+1}.$$

Since  $c(s^t, \xi^t)$  is independent of  $\xi^t$  for all households with  $z(s^t, \xi^t) = 1$ , it follows that  $c(s^{t+1}, \xi^{t+1})$  is not a function of  $\xi^t$ . Strict monotonicity of the value function, alongside strict concavity of the utility function, requires that  $M(s^t, \xi^t)$  be independent of  $\xi^t$ , which in turn implies the same for  $A(s^t, \xi^t)$ . ■

Our final result shows that households follow threshold cost policies with respect to the costs of accessing their brokerage accounts

**Lemma 4** *For any  $(s^t, \xi^{t-1})$ ,  $Z = \{\xi_t | z(s^t, \xi^t) = 1\}$  is a convex set bounded below by 0.*

**Proof.** Assume not, let  $\tilde{Z} = \{\xi_t | z(s^t, \xi^t) = 1\}$  and  $m_{\tilde{Z}} = \int_{\tilde{Z}} h(\xi) d\xi$ . Define  $\bar{\xi}$  implicitly using the equation  $\int_0^{\bar{\xi}} h(\xi) d\xi = m_{\tilde{Z}}$ . By the Axiom of Choice there exists a 1-1 function  $S: \tilde{Z} \rightarrow [0, \bar{\xi}]$ . Given  $B(s^t)$  and  $M(s^{t-1}, \xi^{t-1})$ , for each  $\xi_0 \in \tilde{Z}$  and  $\xi = S(\xi_0)$ , construct an alternate continuation plan that assigns the original continuation plan associated with  $\xi_0$  to  $\xi$ . In other words,  $c^a(s^t, \xi^{t-1}, \xi) = c(s^t, \xi^{t-1}, \xi_0)$ ,  $A^a(s^t, \xi^{t-1}, \xi) = A(s^t, \xi^{t-1}, \xi_0)$  and so on. This alternative plan satisfies (3) - (6) and is thus feasible. Moreover, by construction, it offers the same expected lifetime utility as the original plan. Any solution to the household's problem, solving (2) subject to (3) - (6), must satisfy (33) with equality. Since the original plan solved the household's problem by assumption, the alternative plan satisfies (33) as a strict inequality since

$$\begin{aligned} & \int_{\xi^{t-1}} q(s^t) h(\xi^{t-1}) \int_0^{\bar{\xi}} (x^a(s^t, \xi^t) + P(s^t) \xi_t) d\xi d\xi^{t-1} \\ & < \int_{\xi^t} q(s^t) h(\xi^t) (x(s^t, \xi^t) + P(s^t) \xi_t) z(s^t, \xi^t) d\xi^t ds^t. \end{aligned}$$

This contradicts optimality of the original plan. ■

FIGURE 1A: Endogenous Segmentation Steady-State (mean duration: 4.818)

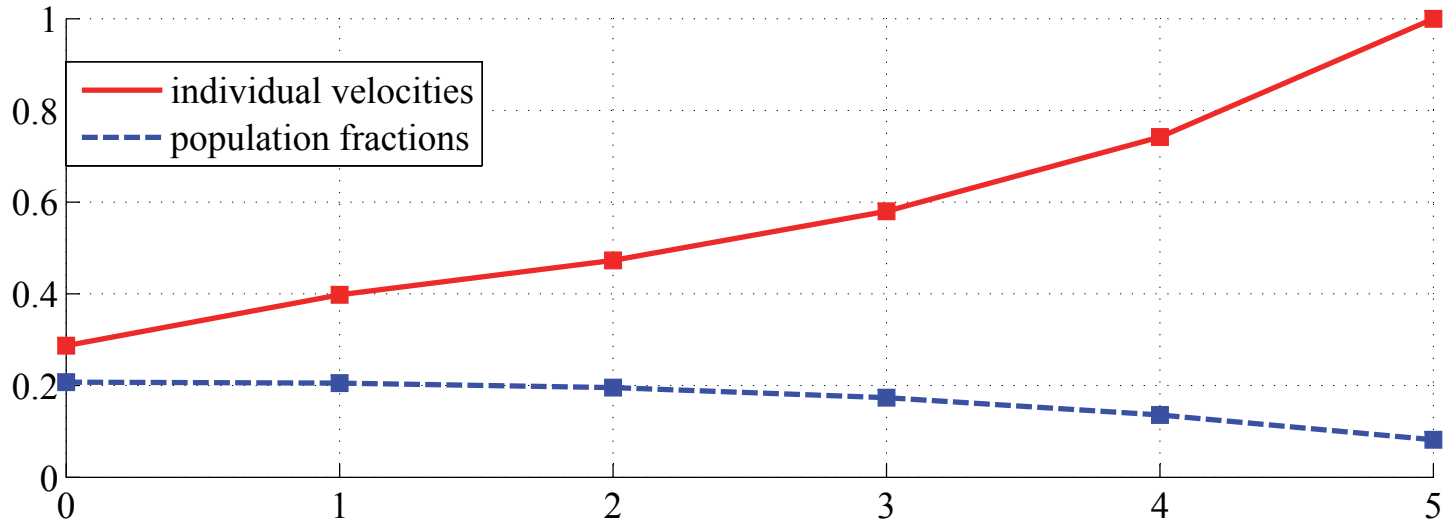


FIGURE 1B: Fixed Duration Steady-State (mean duration: 5)

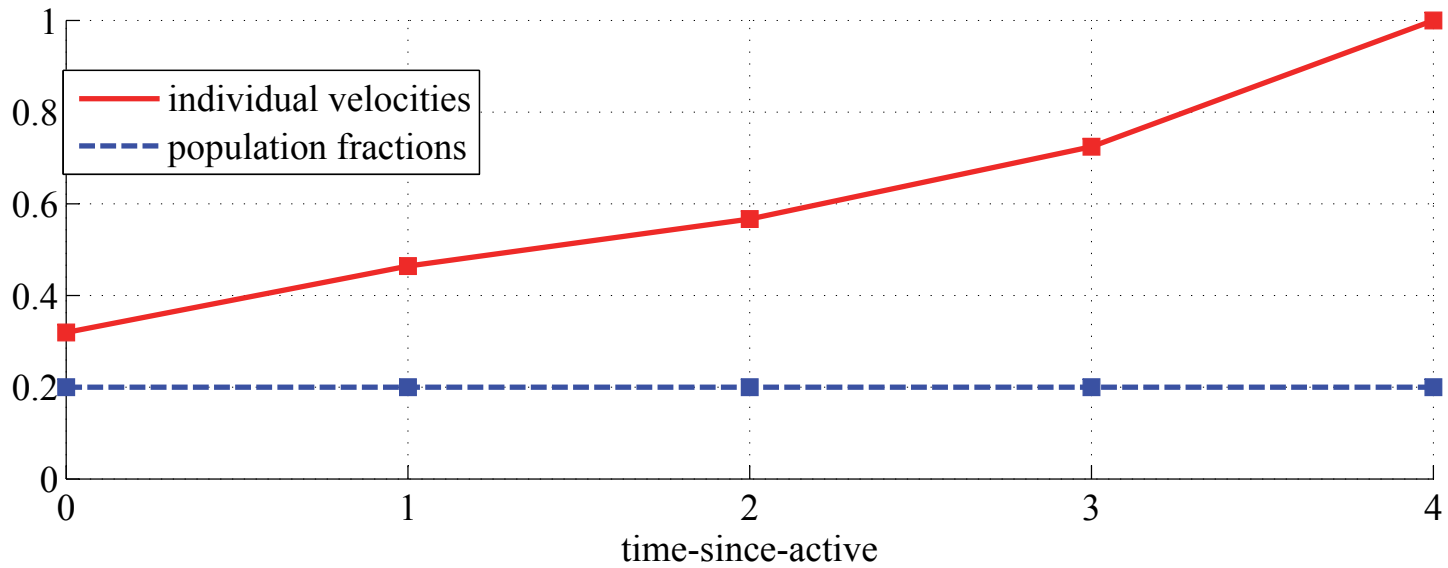


FIGURE 2: Temporary Money Growth Shock in 5-Period Fixed Duration Model

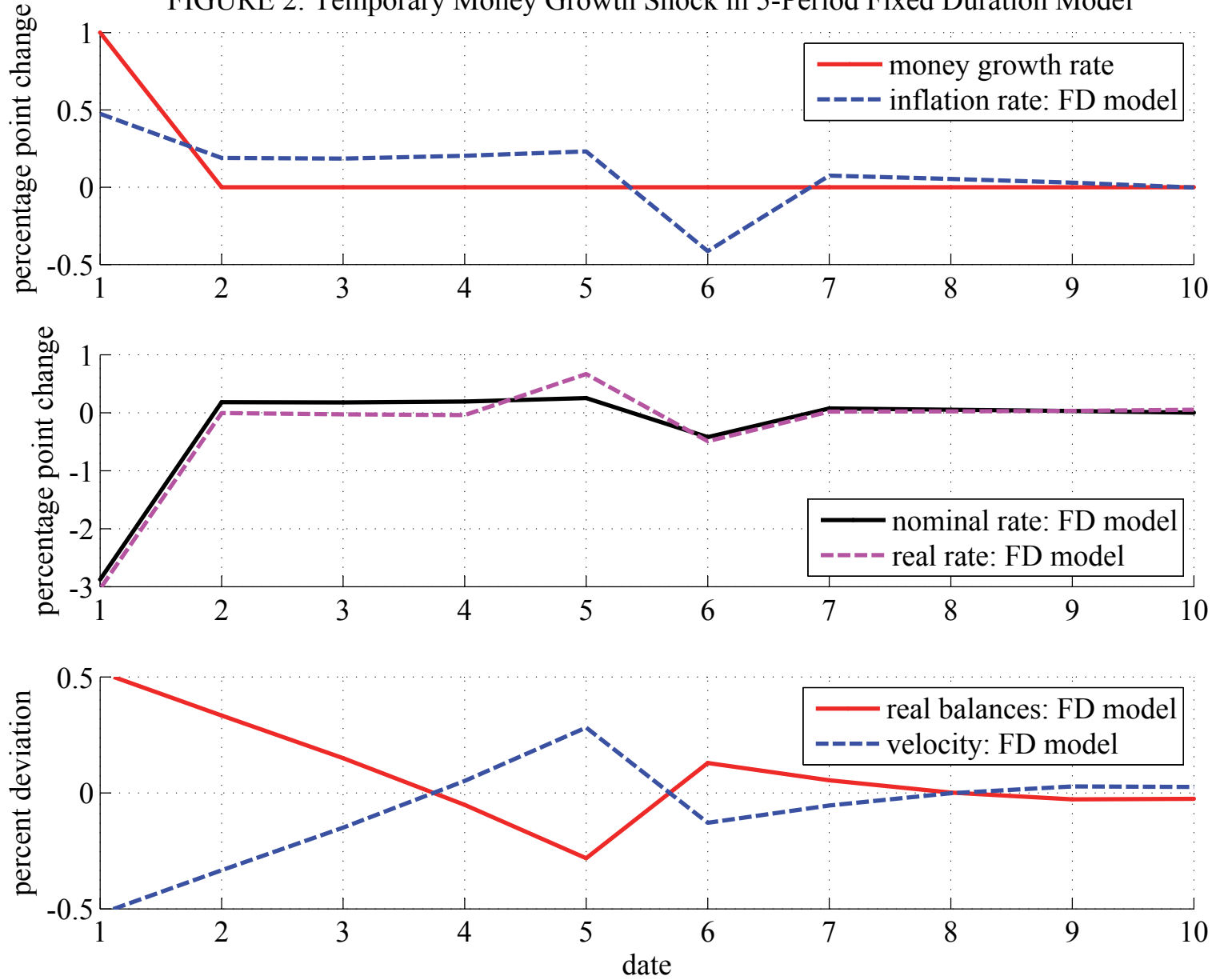


FIGURE 3A: Inflation Rate Responses to Money Growth Shock

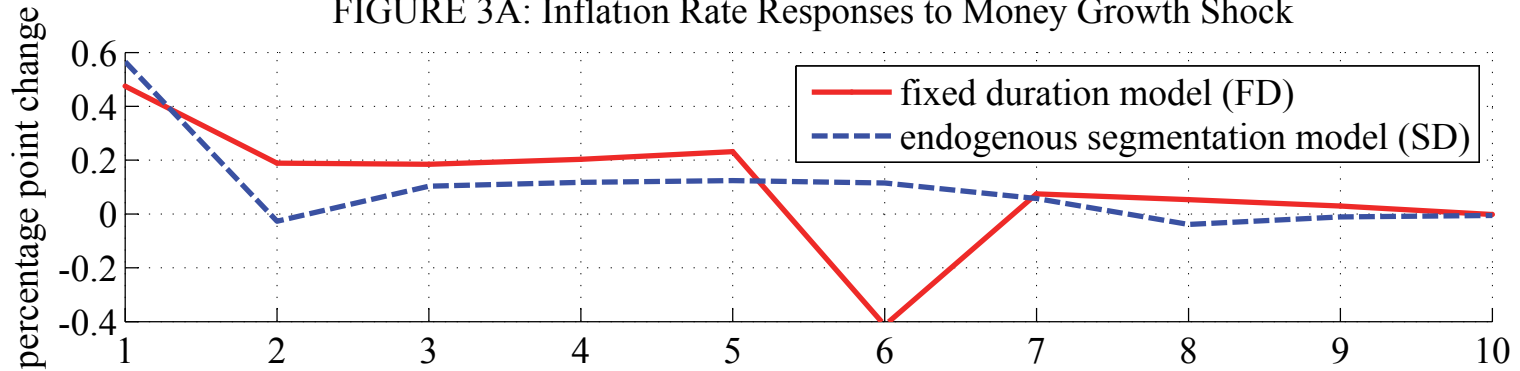


FIGURE 3B: Interest Rate Responses in the Endogenous Segmentation Model

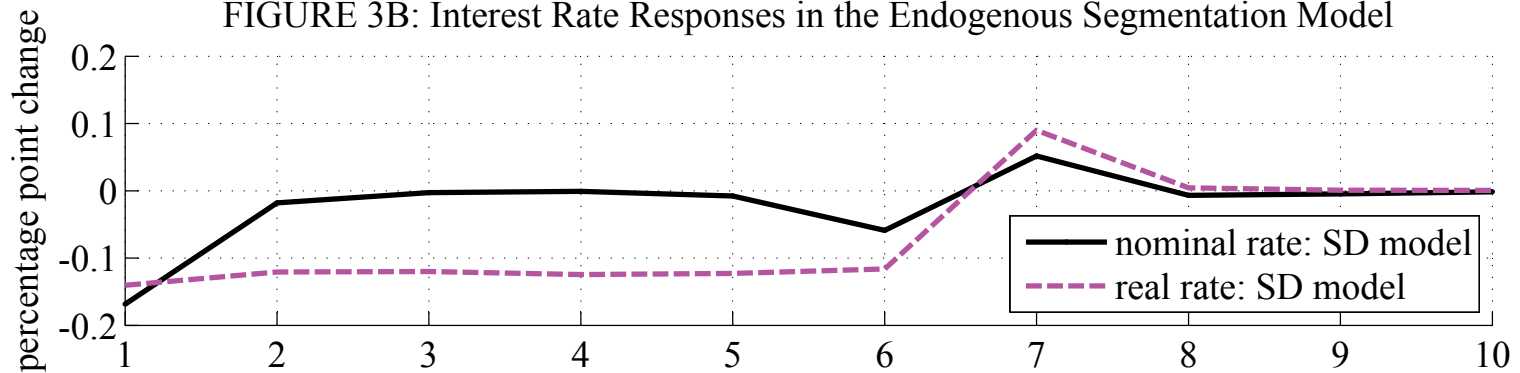


FIGURE 3C: Aggregate Velocity Responses to Money Growth Shock

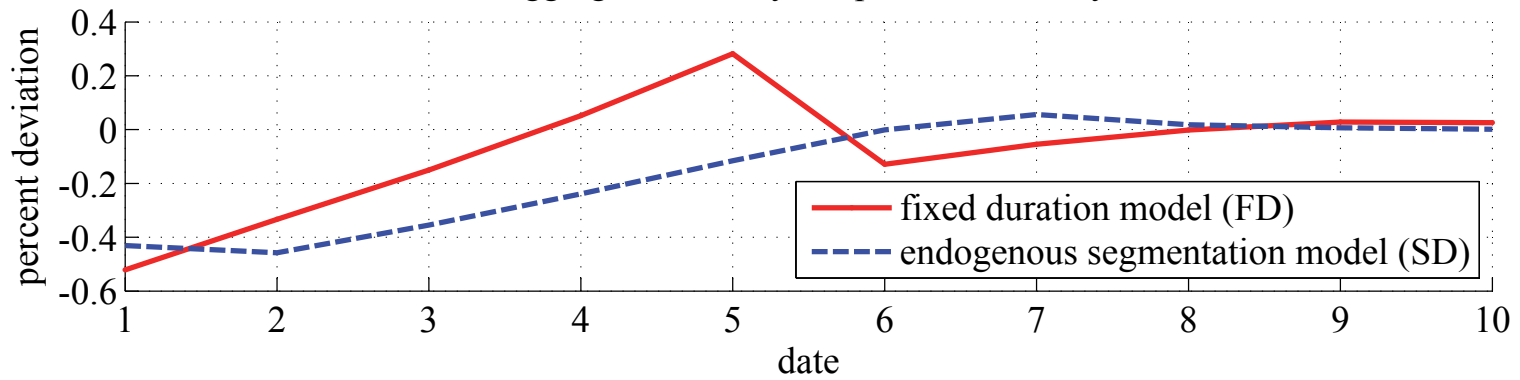


FIGURE 4A: Endogenous Segmentation Response in Total Active Households

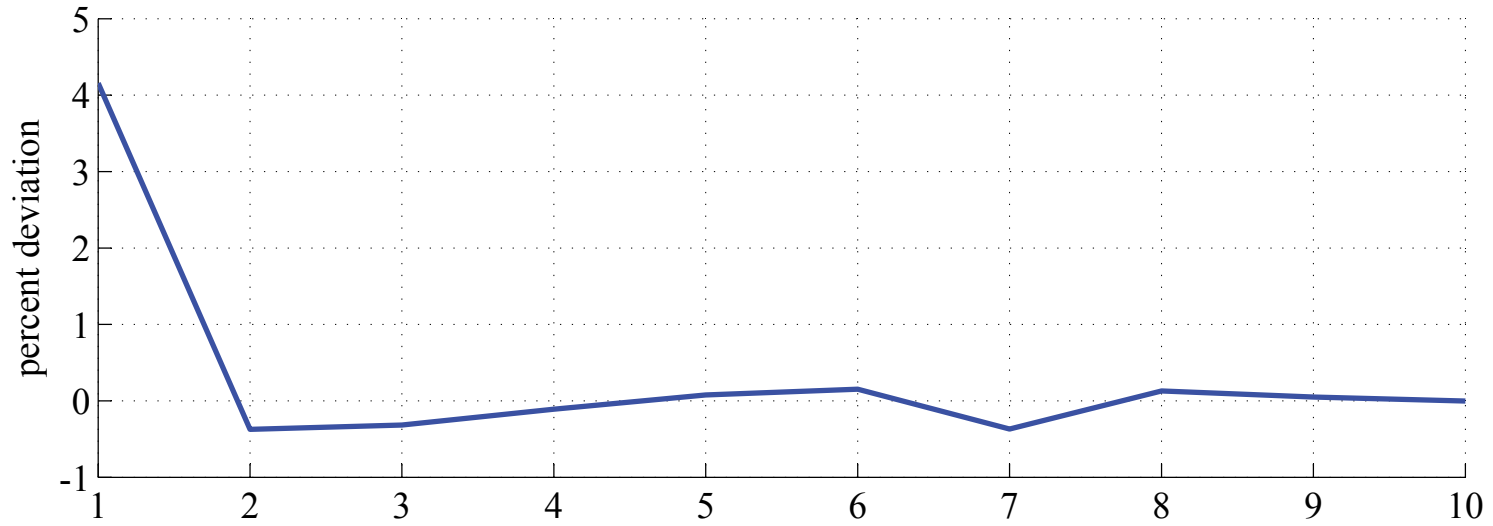


FIGURE 4B: Endogenous Segmentation Response in Consumption of Active Households

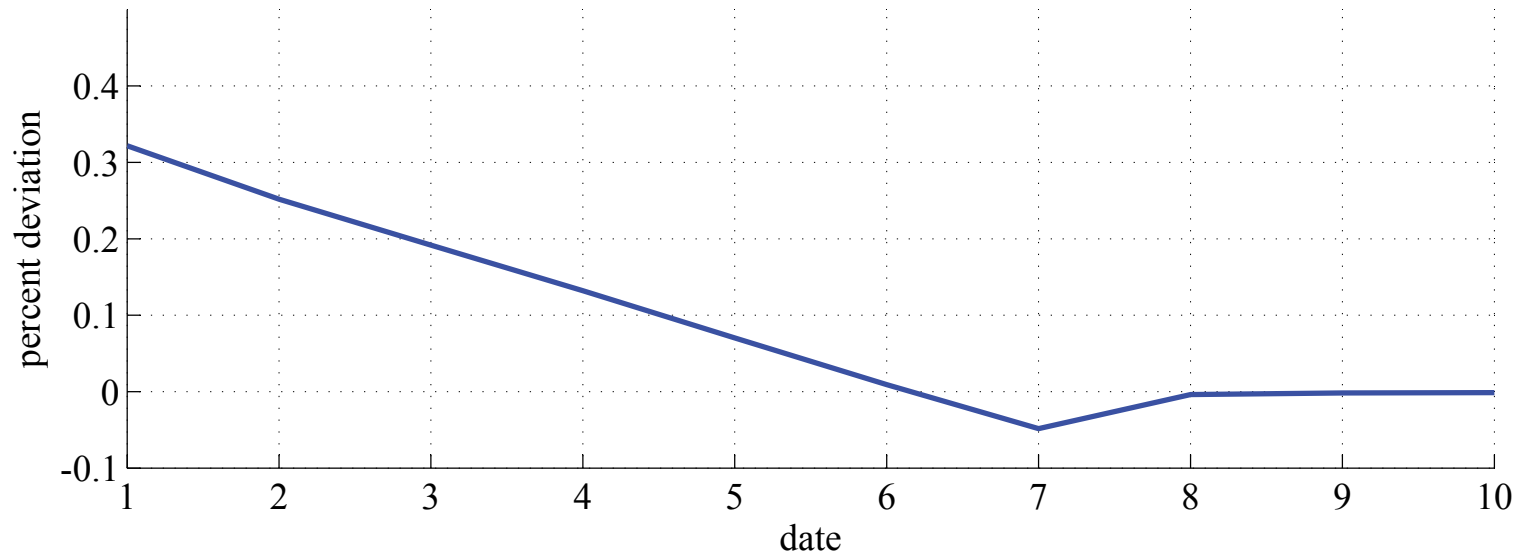


FIGURE 5A: Role of State-Dependent Activity Rates in Inflation Rate Response

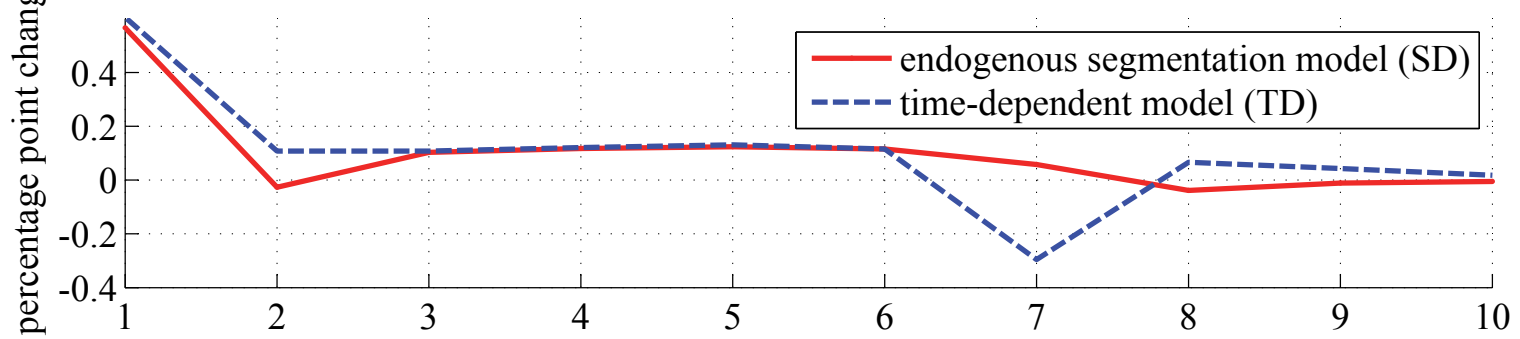


FIGURE 5B: Interest Rate Responses with Changes in Activity Rates Suppressed

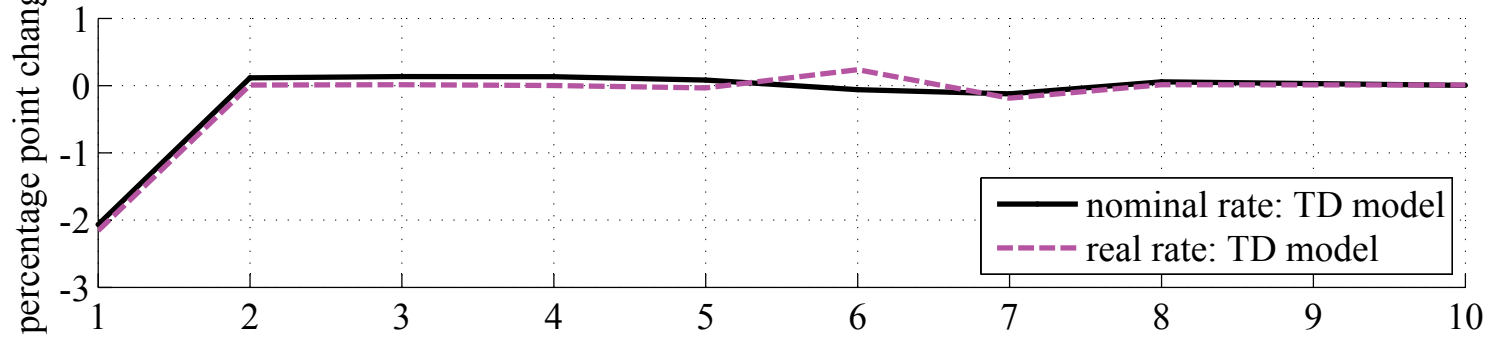


FIGURE 5C: Role of State-Dependent Activity Rates in Velocity Response

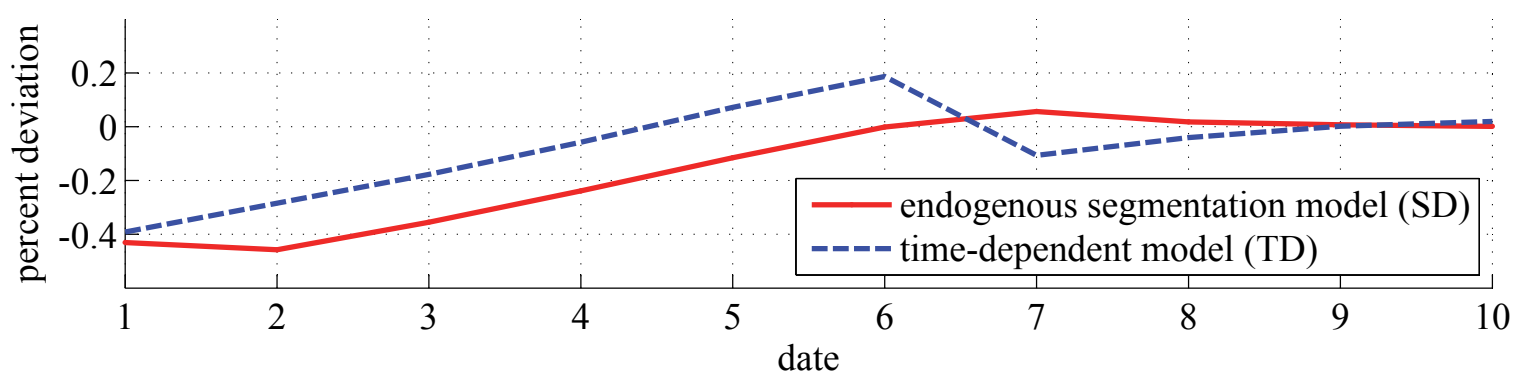


FIGURE 6: Temporary Money Growth Shock in High Mean Duration Model

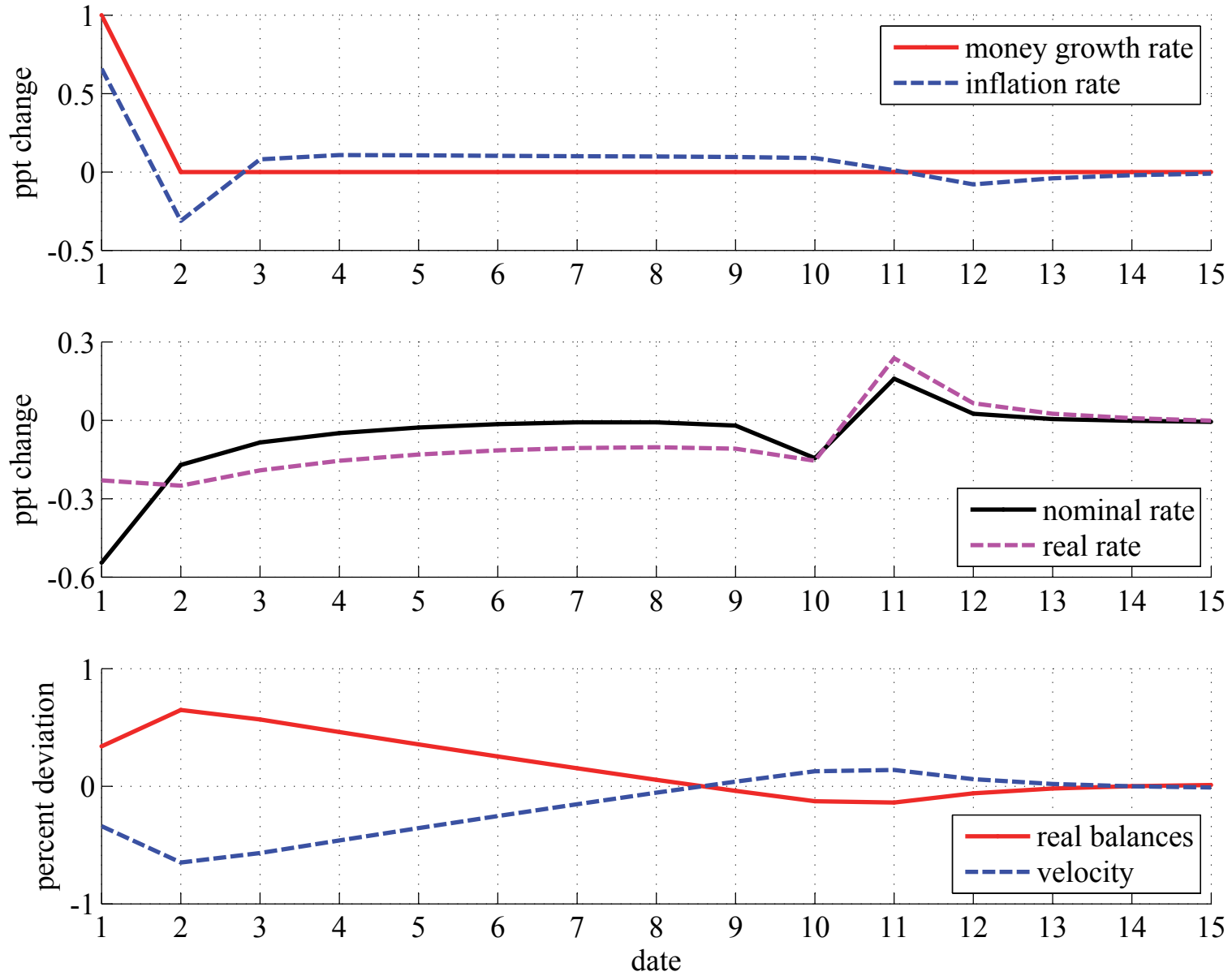


FIGURE 7: Market versus Euler-Implied Real Interest Rates in Our Economy

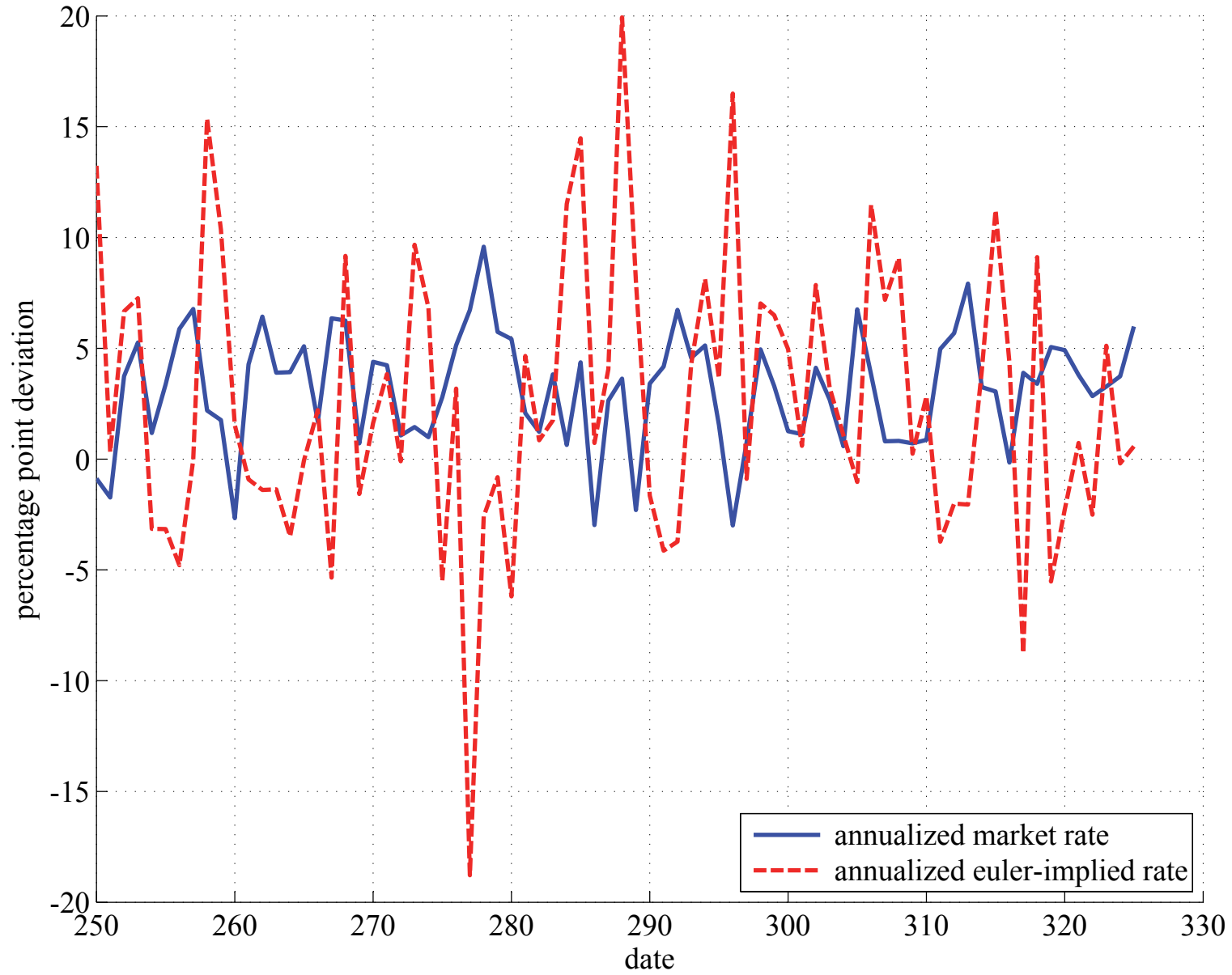
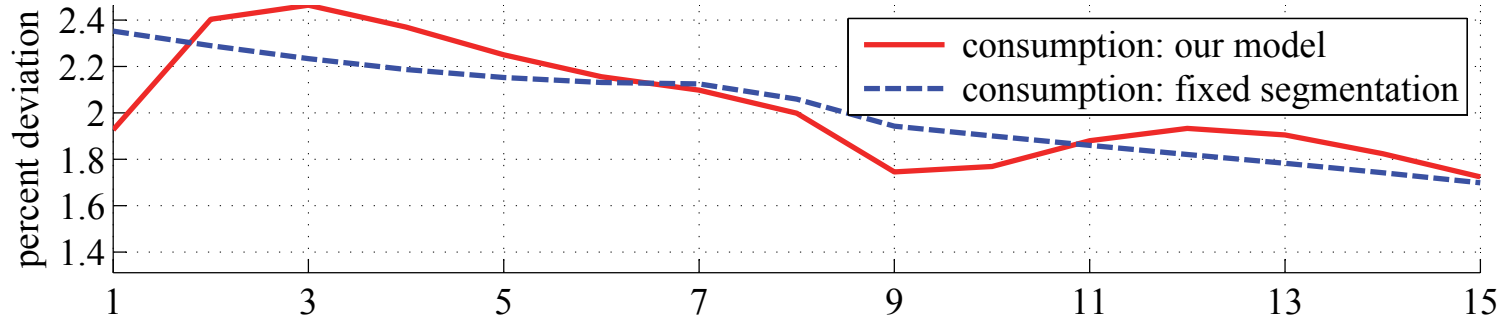
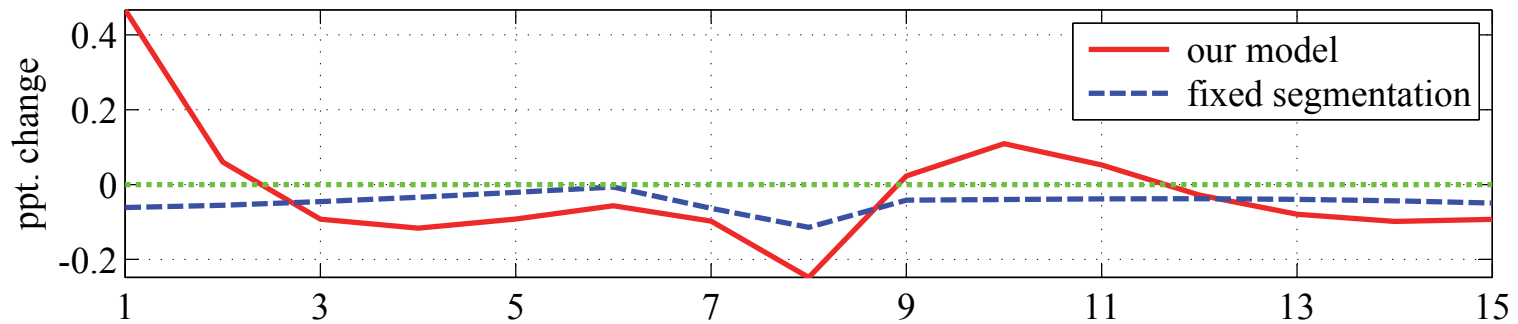




FIGURE 8: A Persistent Positive Productivity Shock



Consumption Growth



Market Real Interest Rate

