Productive Misallocation and International Transmission of Credit Shocks

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ABSTRACT

We develop an asymmetric two-country equilibrium business cycle model to study the role of international trade in transmitting the real effects of financial shocks across borders. Our heterogeneous firms have differing needs for external finance and face occasionally binding collateral constraints hindering their investments, while input-output linkages facilitate trade in both final goods and intermediate inputs. Our model features asymmetric transmission of aggregate shocks between a large economy calibrated to the United States and a small open economy calibrated to Canada, and qualitatively distinct propagation of real versus financial shocks. When confronted with global financial shocks, it predicts that the recession in the large economy appreciably alters the recession in its smaller trade partner, with unique investment dynamics driving the transmission. The reverse does not hold.

Keywords: credit crisis, cross-country shock propagation, capital misallocation

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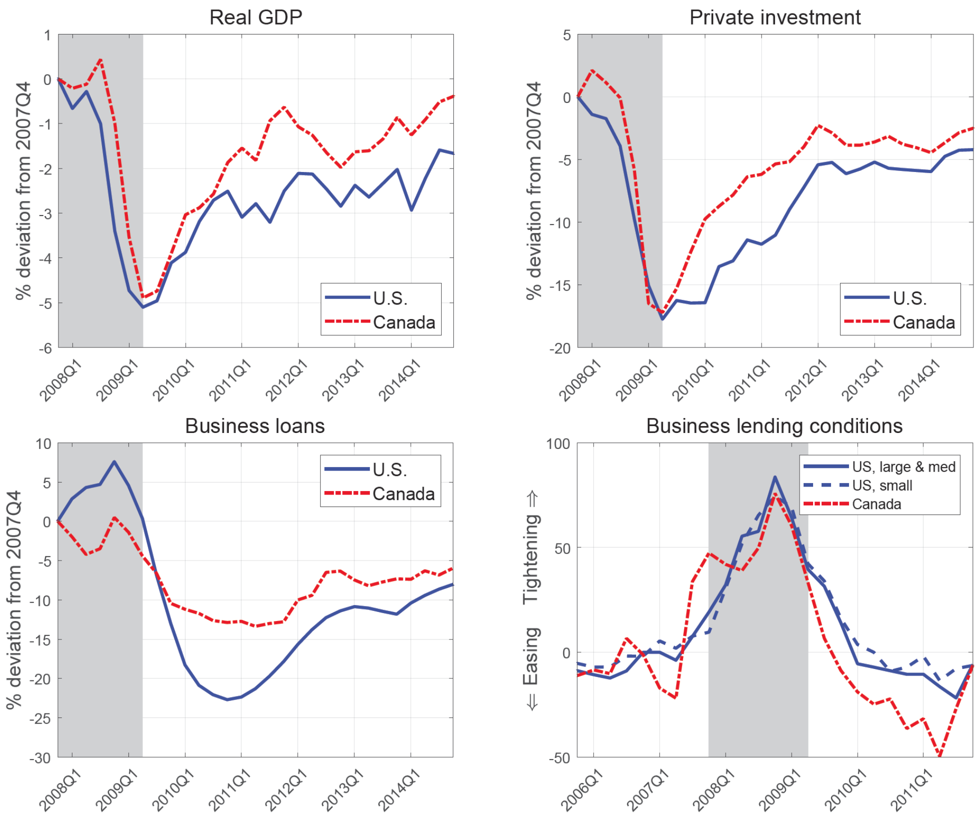
1 Introduction

United States domestic credit market disruptions spread rapidly during the 2007-09 financial crisis, triggering sharp contractions in real activity across advanced and emerging economies. As financial and economic conditions deteriorated, countries throughout the world exhibited a high degree of cross-country synchronization in aggregate quantity variables.[1] Figure 1 demonstrates this comovement in the case of Canada, the leading trade partner for the U.S. at the time.

FIGURE 1. U.S. and Canada over the global financial crisis



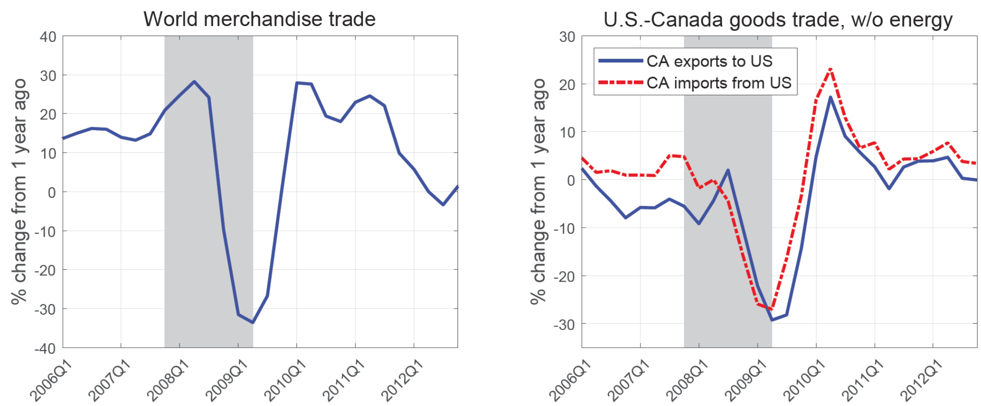
NOTE.– Real GDP and private investment (fixed investment plus consumer durables expenditure) are from the U.S. Bureau of Economic Analysis and Statistics Canada GDP tables; gray bars reflect the U.S. 2007 recession dates. Real business loans series are constructed from United States Board of Governors data on commercial and industrial loans at all commercial banks and Bank of Canada data on loans to non-financial corporations and unincorporated businesses by chartered banks. All quantity series are in logs, detrended using an HP filter with weight 1600. Lending conditions series are the net percentage of banks reporting tightened loan standards in the U.S. Board of Governors' Senior Loan Officer Opinion Survey on Bank Lending Practices and the Bank of Canada' Senior Loan Officer Survey.

While Canada avoided a banking crisis and experienced a lesser fall in business loans, its decline in lending was roughly coincident with that seen in the U.S., as was its tightening of loan standards. The 2008 Canadian recession was less severe and shorter by comparison to many other advanced

[1] Kollmann, Enders and Müller (2011), Ueda (2012), and Perri and Quadrini (2018) provide further evidence of synchronization in real and financial aggregates between the U.S. and advanced economies during the global crisis.

economies, but similar in magnitude and timing to that experienced in the United States. A number of empirical studies emerged from the global financial crisis, uncovering evidence on the nature of shocks originating in financial markets and their severe consequences. Theoretical analyses of these shocks have explored various frictions to isolate propagation mechanisms through which credit disturbances might generate such large economic downturns as seen over this episode.

FIGURE 2. International trade over the global financial crisis



NOTE.– World merchandise trade series is from IMF Direction of Trade Statistics. Bilateral trade series are Canada-U.S. nominal merchandise exports and imports of non-energy products deflated by export and import price indices; these data are from Statistics Canada.

The global recession was accompanied by an abrupt collapse in international trade, as seen in figure 2. World merchandise trade fell 38.3 percent in nominal terms between 2008Q2 and 2009Q1 according to International Monetary Fund Direction of Trade Statistics.[2] Meanwhile, bilateral trade between the U.S. and Canada contracted sharply; Canada's real exports of non-energy goods to the U.S. fell by 29.3 percent between 2008Q2 and 2009Q2, and its real imports of non-energy goods from the U.S. fell 31.4 percent between 2008Q2 and 2009Q1. Some empirical studies link this unprecedented contraction in international trade to fallout from the global financial crisis. Bems, Johnson and Yi (2010), Eaton, Kortum, Neiman and Romalis (2016) and Bussière et al. (2013), among others, have ascribed the trade collapse to large reductions in expenditures on traded goods.[3]

Taken together, many studies related to figure 1 present a compelling case that financial linkages were important in propagating economic distress across countries. At the same time, the great concurrent global trade decline in figure 2 suggests that trade linkages also may have had a role in transmitting negative consequences of the U.S. financial crisis across the world. This paper uses an

[2] Bems, Johnson and Yi (2013) offer an excellent survey of works examining the collapse in trade during the crisis.

[3] Behrens, Corcos and Mion (2013) attribute more than half of the fall in Belgian firms' exports to reductions in their destination economies' GDP growth rates.

equilibrium international business cycle model to examine trade linkages as a contributing factor in propagating the real consequences of financial disruptions from one country to its trade partners.

We extend the Khan and Thomas (2013) model, wherein firms heterogeneous in capital and financial position face persistent idiosyncratic productivity shocks alongside occasionally binding borrowing limits, to an open economy setting with two asymmetric countries. Firms' investments are financed with internal funds and one-period debt, but their debts are limited by a collateral constraint anchored on their existing capital stocks. Exogenous death and birth ensure there are always some firms in each country that cannot borrow what they need to reach the efficient capital stocks consistent with their productivities. The resulting capital misallocation lowers aggregate productivity, employment, investment and GDP even in ordinary times, and it is through this channel that a credit shock tightening collateral constraints causes a domestic recession.

While our firms are largely patterned on those in Khan and Thomas (2013), we introduce a direct interdependency between their production and trade in line with salient regularities reported in the recent trade literature. Cross-country production linkages have been emphasized there as an increasingly prominent aspect of international business cycles, with intermediate inputs now accounting for a sizeable share of international trade.[4] With that in mind, we assume a roundabout structure whereby firms' output is used as both final goods and production inputs, at home and abroad, and ensure that our calibrated model captures the extent of intermediate inputs use relative to firms' value added, as well as the share of imported content in these inputs.

We calibrate our economy to the United States and Canada, replicating each country's trade shares and relative production sizes. The strong trade relationship between these two countries, alongside their geographic proximity, makes this pairing a natural candidate for an analysis focused on the role of bilateral trade linkages in the transmission of aggregate financial shocks.[5] Our calibrated model reflects the differences in these countries' trade reliance, with the U.S. having minimal exposure to the Canadian economy relative to the size of its domestic activity and Canada having substantially greater exposure to the U.S. economy relative to its domestic activity. As such, it is a useful laboratory to study the transmission of a U.S. financial shock to its smaller individual trading partners and vice versa. A further advantage of calibrating to these two advanced neighboring economies is that it avoids some considerations absent from our model that themselves may have non-trivial quantitative effects on cross-country shock transmission through trade, in

[4] See, for example, Burstein, Kurz and Tesar (2008) and di Giovanni and Levchenko (2010).

[5] The World Bank's World Integrated Trade Solution data show Canada was the top destination for U.S. exports in 2007, accounting for 21.4 percent of total U.S. exports, and it was the second largest source of imports into the U.S. (15.7 percent), closely following China (16.9 percent). At the same time, 79 percent of Canadian exports went to the U.S., and the U.S. supplied 54.2 percent of Canada's imports.

particular, exchange rate policy, trade policy and transportation costs and lags.

We begin our analysis by studying responses to tightened collateral constraints in our model's larger economy calibrated to the United States, setting the shock's size and persistence so that the predicted path of debt matches that of U.S. nonfinancial business debt over the financial crisis. As in Khan and Thomas (2013), the domestic result is a growing distortion in the allocation of capital that drives a persistent decline in aggregate productivity, and hence in aggregate investment, employment and production. Examining firm-level consequences of the shock, we demonstrate that disruptions to young cohorts' life-cycle growth lie at the heart of the nonmonotone aggregate TFP response shaping the domestic recession. Born comparatively small, young firms tend to be cash-poor and so more reliant on external funds than older firms. Tight credit increases the inefficiency in their investments relative to other firms' and extends their maturing phase. Since all cohorts born during the crisis see their life-cycle growth protracted, a growing number are affected, worsening overall misallocation. That in turn prolongs the drag on TFP beyond the restoration of credit conditions, rendering the recoveries in aggregate investment and other series unusually gradual.

As it persistently lowers the relative return to aggregate capital in our large economy, the financial shock is transmitted through an investment channel to its trade partner. Responding to the wealth effect implied by their worsening recession, large-economy households reduce their savings by less than the decline in domestic investment, achieving this with changes in net goods flows abroad. Effectively, they relocate some savings to the economy where financial conditions have in relative terms improved, a shift mirrored by a persistent rise in their country's trade balance and a temporary worsening in its terms of trade. These adjustments have a non-trivial impact on our small economy, where the external sector accounts for a sizable share of real activity. There, a markedly greater rise in investment than in domestic savings ramps up aggregate capital to accommodate an increasing demand for domestic production in anticipation of rising costs of imported goods. The result is a modest expansion that lasts until the credit situation abroad begins to improve. At that point, global production starts slowly shifting back towards the large economy, and investment in the small economy begins a correspondingly slow decline. This protracted investment reversal drives a long contraction in the small economy, with reductions in investment contributing over 40 percent of the ultimate GDP losses, and a markedly sluggish recovery thereafter.

Aggregate shock transmission is asymmetric in our model given its calibration to the U.S., a vast economy with negligible trade shares, and Canada, a far smaller economy heavily reliant on trade. A financial shock to the small economy induces investment relocations and net export dynamics qualitatively similar to those discussed above. However, given its far larger productive scale and limited exposure to trade, real activity in the large economy is almost entirely unaffected by them. This prediction of our model is consistent with findings in the empirical literature that business cycle synchronization correlates with the strength of trade linkages across countries.

The type of shock also matters. Large changes in investment are key to credit shock transmission at home and abroad; tight credit steadily erodes the return to domestic capital accumulation by disparately affecting cohorts needing it most, raising relative returns to investing abroad. By contrast, a persistent productivity shock scales domestic firms' production down evenly, and it does so immediately. This implies no rise in misallocation, so no further erosion in TFP, and it implies an immediate rise in import prices for its trading partner. Thus, the small economy also enters recession right away, despite a slight initial rise in investment. Expenditure shares of GDP in both countries are roughly unaltered by the shock; thus, changes in consumption expenditures account for the bulk of their GDP responses.

Finally, we explore an exercise featuring direct credit shocks to both economies, with the second shock set to match the fall in Canadian business debt over the global financial crisis. There, the small economy's recession is altered in three ways by its trading partner's deep financial recession, all traceable to one source. Because its shock is smaller and its economy is more exposed to trade, it receives infusions of savings from abroad at the start of the crisis, propping its investment up at a time when growing misallocation is lowering the return to aggregate capital. Those net goods flows soften the initial downturn, appreciably delay the recession trough, and extend the half-life of its GDP recovery by more than a decade. Comparing results from our model's global financial crisis to the U.S.-Canada experience over this episode, we see similarities in the coincident start of their downturns, Canada's small initial investment rise and sizeable declines in its trade balance with the U.S. starting in 2008, as well as the unusual sluggishness in both countries' economic recoveries. The greatest differences lie in the far smaller magnitudes of decline in our model, and the fact that the Canadian recovery began alongside that in the United States and proceeded at a similar rate.

The remainder of the paper is organized as follows. Section 2 reviews the literature related to our study. We describe the model economy in Section 3 and discuss its calibration in section 4. Section 5 presents our quantitative results, and section 6 concludes.

2 Related literature

Our analysis of the economic consequences of financial shocks builds on advancements in the closed-economy quantitative financial frictions literature prompted by the U.S. 2007 recession. We particularly draw on the work of Khan and Thomas (2013), extending their collateral constraint model to a two-country business cycle setting with input-output linkages capturing international trade in both final goods and intermediate inputs.[6] Reconsidering the mechanics underlying the nonmonotone TFP response to a domestic tightening of borrowing constraints in this environment, we show that tight credit is particularly damaging to the life-cycle growth of young firms, so capital misallocation worsens as the number of affected cohorts expands, discouraging domestic investment.[7] Considering effects abroad, we find the investment channel is also central to the international transmission of a financial shock due to a temporary rise in the relative return to aggregate capital accumulation for the economy experiencing no direct shock.

As investment contractions are central to financial shock propagation in our generalization of the Khan and Thomas (2013) model, our work complements other studies emphasizing alternative mechanisms through which firms' financing needs propagate aggregate credit disturbances. One such mechanism operates through the labor demand channel. Arellano, Bai and Kehoe's (2019) firms use defaultable debt to finance their wage bills before observing idiosyncratic productivities, so an uncertainty shock raising default risk discourages hiring through raised loan interest rates. In Jermann and Quadrini (2012), a representative firm faces a binding enforcement constraint limiting working capital loans, so its effective cost of labor rises when a shock tightens that constraint.[8] Another body of work emphasizes the role of trade credit in transmitting financial shocks along production chains by disrupting interfirm borrowing used to finance intermediate inputs. In Luo (2020) and Antinoglu (2021), firms borrow from banks and intermediate input suppliers to pay their wage bills and input costs before production, and the financial interdependency between firms amplifies the aggregate impact of shocks to bank lending costs or borrowing constraints. Bocola and Bornstein (2023) and Reischer (2024) endogenize the use of trade credit in multi-sector models and show its disruption acted as a credit multiplier significantly amplifying the U.S. 2007 recession.

Our focus on financial shock transmission through international goods trade also complements a literature exploring financial contagion. Devereux and Sutherland (2011), Dedola and Lombardo (2012), Perri and Quadrini (2018), Yao (2019) and Devereux and Yu (2020) examine models where investors access foreign assets subject to financial frictions. Given direct exposure to foreign assets in

[6]Collateral constraints have been widely adopted in the closed-economy financial frictions literature since Kiyotaki and Moore's (1997) theoretical work showing they propagate shocks to the value of collateral. A popular alternative, Bernanke, Gertler and Gilchrist's (1999) financial accelerator mechanism, derives from Townsend's (1979) costly state verification model where information asymmetries yield borrowing premia inversely related to net worth.

[7]Khan and Thomas (2013) argue this point based on disproportionate employment losses among small firms and the correlation between age and size. We establish it directly using moments from the firm age-size distribution.

[8]The credit-induced distortion in Buera and Moll (2015) may be an efficiency (TFP), investment or labor wedge depending on whether idiosyncratic shocks affect firms' productivities, investment costs or labor recruitment costs.

these settings, a financial shock in one country worsens borrowing conditions abroad, inducing cross-country comovement in investment and production. Other studies allow an explicit transmission role for global banks; see, for example, Mendoza and Quadrini (2010), Kollmann, Enders and Müller (2011), Ueda (2012) and Beaton, Lalonde and Snudden (2014).[9] We do not model financial contagion, but allow for it via simultaneous shocks. Comparing responses to a domestic credit tightening in isolation versus alongside a financial recession in its large trading partner, we find that the trade channel reshapes a small open economy's financial recession in noteworthy ways.

More broadly, our comparisons of financial versus real shock transmission in an asymmetric, two-country model contribute to an extensive literature studying aggregate shock transmission in international business cycle models beginning with the work of Backus, Kehoe and Kydland (1992, 1994) and Baxter and Crucini (1995). Whereas early work featured symmetric, one-sector models driven by productivity shocks, subsequent studies have replaced one or more of these assumptions to strengthen the framework's internal propagation and in other ways improve its empirical performance. Kose and Yi (2006) develop a three-country business cycle model allowing differing Armington aggregator weights and productivity shock processes; they find it succeeds in generating a positive relation between trade linkages and output comovement, though the implied elasticity is weak in comparison to its empirical counterpart. While we do not quantify this relationship in our analysis, our results complement theirs in demonstrating that an economy more reliant on international trade in proportion to its GDP is more susceptible to financial shocks abroad.

Miyamoto and Nguyen (2017) emphasize labor market responsiveness to relative price changes as a key factor in strengthening business cycle transmission. Using a model estimated with U.S. and Canadian data, they show preferences yielding a low wealth elasticity of labor supply, imported intermediate inputs, and variable capital utilization each help boost transmission of permanent productivity shocks, and together deliver substantial comovement across countries. Our U.S. and Canada model does not include variable utilization; however, the other two elements strengthen its transmission of a productivity shock recession across borders, consistent with their findings.

Alessandria, Kaboski and Midrigan (2013) find the inclusion of inventories is successful in addressing several deficiencies of the standard model. Embedding a stockout-avoidance motive in the retail sector of their two-country business cycle model, they show that stock adjustments amplify the volatility of international trade flows following aggregate productivity shocks, and help generate countercyclical real net exports while reducing the consumption correlation across countries. We do

[9] See also Kalemli-Ozcan, Papaioannou and Perri (2013), Kamber and Thoenissen (2013) and Kollmann (2013). As in the closed-economy literature, earlier work by Gilchrist (2004), Iacoviello and Minetti (2006), Faia (2007), and Devereux and Yetman (2010) studies real and nominal shocks in two-country models with financial frictions.

not undertake stochastic simulations to compute business cycle moments from our model; however, its impulse responses show that GDP and net exports commove negatively in both countries when either experiences a financial shock. Whereas the countercyclicality of net exports stems from procyclical inventory accumulation in Alessandria, Kaboski and Midrigan (2013), it instead reflects the relocation of aggregate investment from an economy experiencing increased financial distortions to a trading partner economy with relatively improved financial conditions in our model, and is thus instrumental in its cross-country transmission of financial shocks.

Finally, the inclusion of input-output linkages in and across countries in our model is prompted by recent work emphasizing the growing importance of trade in intermediate inputs, particularly in light of their contribution to the trade collapse during the global financial crisis.[10] Embedding a vertically integrated production structure in their two-country real business cycle model, Burstein, Kurz and Tesar (2008) find cross-country production linkages and complementarity in domestic and foreign inputs strengthen the relationship between trade and business cycle comovement. Johnson (2014) introduces production linkages in a multi-country model, and shows it predicts the strong correlation between trade linkages and output comovement only when shocks are correlated across countries. Elsewhere, Alessandria and Choi (2014) find these linkages contribute to the gains from trade in a model where sunk and flow fixed costs influence firms' forward-looking choices on whether to enter or remain in export markets. Consistent with the results in these earlier studies, we find allowing a realistic share for material inputs in our model increases cross-country transmission of productivity shocks; however, it is of little consequence in the transmission of financial shocks.

3 Model

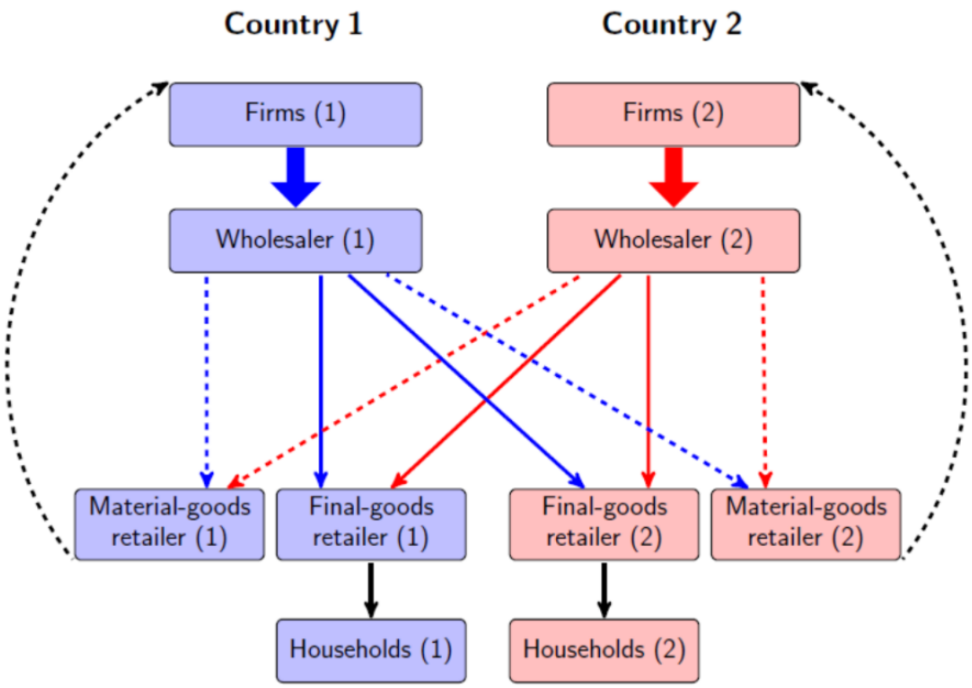
We consider two countries, $c = 1, 2$, distinguished in size by their time-invariant measures of households, Ψ_c. The countries are symmetric in most other respects, though their differing sizes imply significant differences in international trade exposure in our calibration. All markets are perfectly competitive, prices are flexible, and we assume complete international financial markets.

Each country houses a set of identical infinitely-lived households with access to state-contingent nominal bonds. The production structure features a form of roundabout production yielding input-output linkages within and across countries. A representative final goods retailer combines domestic and imported inputs to produce the good used for home consumption and capital investment. A

[10] Bems, Johnson and Yi (2011) report that trade in intermediate inputs accounted for over 40 percent of the fall in total trade during this episode. Levchenko, Lewis and Tesar (2010) and Bems, Johnson and Yi (2010) also demonstrate the empirically important role of intermediate inputs in the trade collapse following the global downturn.

wholesaler intermediates the sale of domestic firms' output to final goods retailers at home and abroad, and to domestic and foreign material goods retailers that, in turn, supply materials to domestic firms. Finally, each country has a time-invariant measure of heterogeneous firms matching its household population size, and each such firm uses capital, labor and materials in its production.

MARKET INTERACTIONS



Firms produce a common domestic good; they face persistent country-specific aggregate total factor productivity shocks and persistent idiosyncratic productivity shocks. Firms hire labor from domestic households, purchase materials from the domestic material goods retailer and maintain their own capital stocks. Each invests goods bought from the domestic final goods retailer to augment its capital for the next period, and each can use one-period loans to help fund its purchases. A country-specific collateralized borrowing constraint limits the debt a firm can take on as a function of its existing capital stock, and firms cannot circumvent the constraint by paying negative dividends. A constant exogenous exit and entry rate each period prevents all firms effectively outgrowing financial frictions in the long run, while maintaining a fixed measure of firms over time.

The aggregate state of the world economy is A, where $A \equiv (S, Z)$ and $Z \equiv [z_1, z_2, \zeta_1, \zeta_2]$ is the exogenous state vector. The latter includes aggregate productivity, z_c, and the credit state ζ_c, in each country, $c = 1, 2$. Each country's ζ_c anchors the collateral constraints limiting firms' debt in proportion to their current capital. All exogenous state variables follow Markov chains; z takes on one of N_z values, ζ takes on one of N_ζ values, and the joint transition matrix Π^Z has elements

$\{\pi^Z_{lm}\}$ representing $pr\{Z'_m|Z_l\}$ for $l,m = 1,\ldots,N_Z$, where $N_Z = N_z N_\zeta$.

Our model generates a time-varying distribution of firms over capital, ($k \in \mathbf{K}\subset\mathbf{R}_+$), debt ($b \in \mathbf{B}\subset\mathbf{R}$) and firm-specific productivity ($\varepsilon \in \mathbf{E}$) in each country. We summarize the distribution of firms at the start of a period in country c using the probability measure μ_c defined on the Borel algebra $\mathcal{S}$ generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K}\times\mathbf{B}\times\mathbf{E}$ for each $c = 1,2$. The model's endogenous aggregate state vector comprises these distributions; $S \equiv [\mu_1,\mu_2]$.[11] All agents take as given the laws of motion determining Z' given Z, as well as the evolution of the endogenous state according to an equilibrium mapping $S' = \Gamma(A)$. We describe the preferences, technologies and optimization problems in each country c and its trading partner $\widetilde{c} \neq c$ below, specifying country 1 or country 2 where necessary for clarity or in defining notation.

3.1 Households

Each of Ψ_c identical households in a country is endowed one unit of time per period, values consumption and labor supply by a period utility function $u(C,N)$ and discounts future utility by the subjective discount factor $\beta \in (0,1)$. Household wealth is composed of one-period shares in domestic firms, identified by the measures ξ_c, noncontingent real bonds corresponding to the debts of domestic firms, denoted by ϕ_c, and aggregate state-contingent nominal bonds. The aggregate bond is denominated in units of country 1 currency, and $B_c(A)$ denotes country c per-capita nominal bonds redeemed in the current period given aggregate state A.

Each household in country 1 chooses its consumption, C_1, the labor hours it supplies to domestic firms, N_1, its shares in firms of each type with which to begin the next period, $\xi'_1(k',b',\varepsilon')$, and its noncontingent real bonds for next period, ϕ'_1. The household also decides its purchases of state-contingent nominal bonds for next period, $B_1(A')$, each guaranteeing one unit of country 1 currency if state A' is realized. Let $\varrho(A';A)$ be the real price of one such bond in units of country 1 final goods, and define $\widetilde{\rho}_1(k,b,\varepsilon;A)$ as the dividend-inclusive values of current firm shares and $\rho_1(k',b',\varepsilon';A)$ as the ex-dividend prices of new shares in a given firm type.

The country 1 real wage and aggregate price level are $w_1(A)$ and $P_1(A)$, respectively, and each real bond therein costs domestic households $q_1(A)$ units of consumption. Finally, let $G(A'|A)$ represent the conditional probability of realizing a given state A' next period, which is determined by $S' = \Gamma(A)$ and the exogenous transition probabilities for the elements of Z. Each country 1 household's expected lifetime utility maximization problem then can be written as follows.

[11] Households' state-contingent bond holdings, $[B_1,B_2]$, are indeterminate in equilibrium.

$$V_1^h(\xi_1, \phi_1, B_1(A); A) = \max_{C_1, N_1, \xi_1', \phi_1', B_1(A')} u(C_1, N_1) + \beta \int V_1^h(\xi_1', \phi_1', B_1(A'); A') G(dA'|A) \quad (1)$$

subject to:

$$\int \widetilde{\rho}_1 (k, b, \varepsilon; A) \, \xi_1 (d [k \times b \times \varepsilon]) + \phi_1 + \frac{B_1(A)}{P_1(A)} + w_1(A) N_1 \geq$$

$$C_1 + \int \rho_1 \left(k', b', \varepsilon'; A\right) \xi_1' \left(d \left[k' \times b' \times \varepsilon'\right]\right) + q_1(A)\phi_1' + \int \varrho(A'; A) B_1(A') dA'$$

Let $\lambda_1(A) = D_1 u(C_1, N_1)$ be the Lagrange multiplier on the budget constraint. Household efficiency conditions with respect to hours worked, shares and real bonds imply restrictions on the country 1 real wage, firm share prices and inverse loan price listed in (2) - (4). Efficiency conditions with respect to state-contingent nominal bonds yield the additional restrictions in equation 5.

$$\begin{aligned} w_1(A) &= -D_2 u(C_1, N_1)/\lambda_1(A) && (2) \\ \rho_1 \left(k', b', \varepsilon'; A\right) &= \int \frac{\beta \lambda_1(A')}{\lambda_1(A)} \widetilde{\rho}_1 \left(k', b', \varepsilon'; A'\right) G(dA'|A) && (3) \\ q_1(A) &= \int \frac{\beta \lambda_1(A')}{\lambda_1(A)} G(dA'|A) && (4) \end{aligned}$$

$$\varrho(A'; A) = \frac{\beta \lambda_1(A')}{\lambda_1(A)} \frac{1}{P_1(A')} G(A'|A) \quad (5)$$

Households in country 2 solve a similar problem adjusted to reflect that state-contingent nominal bonds are not denominated in their currency. Let $Q(A)$ represent the current real exchange rate, defined as the price of country 2 final goods in units of country 1 final goods. Each such bond held at the start of the period returns $\frac{1}{P_1(A)}$ units of country 1 final goods, each worth $\frac{1}{Q(A)}$ units of country 2 final goods. Similarly, a bond for next period state A' costs $\varrho(A'; A)$ units of country 1 final goods, each worth $Q(A)^{-1}$ units of country 2 consumption.

$$V_2^h(\xi_2, \phi_2, B_2(A); A) = \max_{C_2, N_2, \xi_2', \phi_2', B_2(A')} u(C_2, N_2) + \beta \int V_2^h(\xi_2', \phi_2', B_2(A'); A') G(dA'|A) \quad (6)$$

subject to:

$$\int \widetilde{\rho}_2 (k, b, \varepsilon; A) \, \xi_2 (d [k \times b \times \varepsilon]) + \phi_2 + \frac{B_2(A)}{P_1(A) Q(A)} + w_2(A) N_2 \geq$$

$$C_2 + \int \rho_2 \left(k', b', \varepsilon'; A\right) \xi_2' \left(d \left[k' \times b' \times \varepsilon'\right]\right) + q_2(A)\phi_2' + \int \frac{\varrho(A'; A)}{Q(A)} B_2(A') dA'$$

Country 2 households' efficiency conditions imply restrictions on $w_2(A)$, $\rho_2 \left(k', b', \varepsilon'; A\right)$ and $q_2(A)$ mirroring (2) - (4), with $\lambda_2(A) = D_1 u(C_2, N_2)$, and restrict bond prices to satisfy (7).

$$\varrho(A'|A) = \frac{\beta \lambda_2(A')}{\lambda_2(A)} \frac{Q(A)}{P_1(A') Q(A')} G(A'|A) \quad (7)$$

Equations (5) and (7) combine to determine the evolution of the real exchange rate across each date and state: $Q(A') = \frac{\lambda_2(A')}{\lambda_1(A')}\frac{\lambda_1(A)Q(A)}{\lambda_2(A)}$. Assuming an initial date zero wherein $\frac{\lambda_1(A^0)Q(A^0)}{\lambda_2(A^0)} = 1$, each period's real exchange rate is a ratio of the marginal utility of consumption in country 2 versus 1.

$$Q(A) = \lambda_2(A)/\lambda_1(A) \tag{8}$$

3.2 Retailers and international trade

Each country has two representative retailers. One uses the outputs of domestic and foreign firms to make final goods; the other uses domestic and imported goods to make materials used by domestic firms. This roundabout technology implies direct concurrent effects of international trade on final goods output and upstream in the supply chain. We summarize the problems of retailers in each country $c = 1, 2$ below, again using the notation $\widetilde{c}$ to represent the country's trade partner.

3.2.1 Final goods retailers

The final goods retailer in country c combines per-capita domestic inputs, h^{Dc}, and exports from country $\widetilde{c}$, $h^{X\widetilde{c}}$, in a CES technology to make per-capita final goods, H_c:

$$H_c(h^{Dc}, h^{X\widetilde{c}}) = \left[\theta_{hc}\left(h^{Dc}\right)^{\frac{\rho-1}{\rho}} + (1-\theta_{hc})\left(h^{X\widetilde{c}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{9}$$

where ρ is the (Armington) elasticity of substitution between domestic and foreign inputs, and θ_{hc} is a country-specific relative weight on domestic inputs (home bias). It sells its output at price $P_c(A)$ to domestic households for consumption and domestic firms for investment.

Traded goods are denominated in the currency of the country where they are sold. Let $P_h^{Dc}(A)$ and $P_h^{X\widetilde{c}}(A)$ be the nominal prices the country c final goods retailer pays for its domestic and foreign inputs, hereafter termed *h-goods*. In real terms, the country c retailer pays $P_h^{Dc}(A)/P_c(A)$ and $P_h^{X\widetilde{c}}(A)/P_c(A)$ domestic final goods for each unit of domestic and foreign h-goods it uses.

The country-c final goods retailer solves (10) given its technology, (9). The resulting per-capita conditional factor demands in (11) combine with (9) to deliver the aggregate price level in (12).[12]

$$\max_{h^{Dc}, h^{X\widetilde{c}}} \Psi_c[P_c(A)H_c(h^{Dc}, h^{X\widetilde{c}}) - P_h^{Dc}(A)h^{Dc} - P^{X\widetilde{c}}(A)h^{X\widetilde{c}}] \tag{10}$$

$$h^{Dc} = (\theta_{hc})^{\rho}\left(P_h^{Dc}/P_c\right)^{-\rho} H_c \quad \text{and} \quad h^{X\widetilde{c}} = (1-\theta_{hc})^{\rho}\left(P_h^{X\widetilde{c}}/P_c\right)^{-\rho} H_c \tag{11}$$

$$P_c(A) = \left[(\theta_{hc})^{\rho}\left(P_h^{Dc}(A)\right)^{1-\rho} + (1-\theta_{hc})^{\rho}\left(P_h^{X\widetilde{c}}(A)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{12}$$

[12] We suppress the state arguments of the pricing functions in (11) and elsewhere below to simplify the equations.

3.2.2 Material goods retailers

Production and trade in the material goods sector is as in the final goods sector, though the home bias parameters may differ there. The material goods retailer in country c uses per-capita domestic inputs, m^{Dc}, and exports from country $\widetilde{c}$, $m^{X\widetilde{c}}$, in the production function below to produce per-capita materials, M_c.

$$M_c(m^{Dc}, m^{X\widetilde{c}}) = \left[\theta_{mc}\left(m^{Dc}\right)^{\frac{\rho-1}{\rho}} + (1-\theta_{mc})\left(m^{X\widetilde{c}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{13}$$

It sells its materials at price $P_c^M(A)$ to local firms and pays $P_m^{Dc}(A)$ and $P_m^{X\widetilde{c}}(A)$, respectively, for its domestic and foreign inputs (hereafter, *m-goods*). Given these nominal prices and its technology, (13), it solves the problem in (14).

$$\max_{m^{Dc}, m^{X\widetilde{c}}} \Psi_c[P_c^M(A)M_c(m^{Dc}, m^{X\widetilde{c}}) - P_m^{Dc}(A)m^{Dc} - P_m^{X\widetilde{c}}(A)m^{X\widetilde{c}}] \tag{14}$$

Combining the resulting per-capita conditional factor demands in (15) with (13), we obtain the nominal price of materials in (16).

$$m^{Dc} = (\theta_{mc})^{\rho}\left(P_m^{Dc}/P_c^M\right)^{-\rho} M_c \quad \text{and} \quad m^{X\widetilde{c}} = (1-\theta_{mc})^{\rho}\left(P_m^{X\widetilde{c}}/P_c^M\right)^{-\rho} M_c \tag{15}$$

$$P_c^M(A) = \left[(\theta_{mc})^{\rho}\left(P_m^{Dc}(A)\right)^{1-\rho} + (1-\theta_{mc})^{\rho}\left(P_m^{X\widetilde{c}}(A)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{16}$$

3.2.3 Trade

Recalling the real exchange rate Q, we assume the *law of one price* prevails: $\frac{P_i^{X1}Q}{P_2} = \frac{P_i^{D1}}{P_1}$ and $\frac{P_i^{X2}}{P_1 Q} = \frac{P_i^{D2}}{P_2}$ for $i = h, m$. Thus, we may express country c's real exports of h-goods as $\frac{P_h^{Dc}}{P_c}\Psi_{\widetilde{c}}h^{Xc}$ in units of domestic final goods, and its real exports of m-goods as $\frac{P_m^{Dc}}{P_c}\Psi_{\widetilde{c}}m^{Xc}$. Similarly, its real imports of h-goods and m-goods are $\frac{P_h^{X\widetilde{c}}}{P_c}\Psi_c h^{X\widetilde{c}}$ and $\frac{P_m^{X\widetilde{c}}}{P_c}\Psi_c m^{X\widetilde{c}}$, respectively. Equation 17 combines these trade aggregates to obtain per-capita net exports for use in each country's real GDP accounts.

$$NX_c = \frac{\Psi_{\widetilde{c}}}{\Psi_c}\left(\frac{P_h^{Dc}}{P_c}h^{Xc} + \frac{P_m^{Dc}}{P_c}m^{Xc}\right) - \left(\frac{P_h^{X\widetilde{c}}}{P_c}h^{X\widetilde{c}} + \frac{P_m^{X\widetilde{c}}}{P_c}m^{X\widetilde{c}}\right) \tag{17}$$

3.3 Wholesalers

We include a representative wholesaler in each country c purely for expositional convenience. This zero-profit intermediary buys domestic firms' output at real price $\omega_c(A)$ to supply h-goods and m-goods to the two retail sectors in each country. It is indifferent between selling at home and abroad in light of the law of one price assumption above.

Denoting its per-capita supply of h-goods and m-goods by W_{hc} and W_{mc}, the wholesaler's budget constraint is: $W_{hc} + W_{mc} \leq Y_c$, where Y_c is the total per-capita output it purchases from domestic firms. The first order conditions arising from its static optimization problem in (18) immediately imply a common real price, ω_c, for domestic h-goods and m-goods. Combining this observation with those from section 3.2.3, we arrive at the equilibrium price restrictions listed in equation 19.

$$\max_{W_{hc},\, W_{mc}} \left[\Psi_c W_{hc}\right] \left(\frac{P_h^{Dc}}{P_c} - \omega_c\right) + \left[\Psi_c W_{mc}\right] \left(\frac{P_m^{Dc}}{P_c} - \omega_c\right) \tag{18}$$

$$\frac{P_i^{D1}}{P_1} = \frac{P_i^{X1} Q}{P_2} = \omega_1 \text{ and } \frac{P_i^{X2}}{P_1 Q} = \frac{P_i^{D2}}{P_2} = \omega_2, \text{ for } i = h, m \tag{19}$$

Each country's terms of trade is defined as the ratio of its real price of imports relative to the real price of its exports. Using the results in equation 19, these are: $ToT_1 = \frac{\omega_2 Q}{\omega_1}$ and $ToT_2 = \frac{\omega_1}{\omega_2 Q}$.

3.4 Firms

Reviewing notation above, country c firms receive real price $\omega_c(A)$ for their output, and pay real wage $w_c(A)$ for labor and real price $\frac{P_c^M(A)}{P_c(A)}$ for materials. Each firm enters a period identified by $s \equiv (k, b, \varepsilon)$, where k and b are the capital and debt levels it selected at the end of last period, and ε is its current persistent idiosyncratic productivity. Positive values of b represent debt; negative values are financial savings. The firm repays its debt (or recoups its savings) and produces with capital, labor and materials in a decreasing returns to scale Cobb-Douglas production function:

$$y(n, m; k, \varepsilon) = z_c \varepsilon k^\alpha n^\nu m^\gamma, \tag{20}$$

where z_c is the aggregate productivity shock in its country, $\alpha \in (0,1)$, $\nu \in (0,1)$, $\gamma \in (0,1)$, and $\alpha + \nu + \gamma < 1$. Like aggregate productivity, firm-specific productivity ε follows a Markov chain; it has N_ε realizations and evolves according to transition probabilities $\varphi_{ij}^\varepsilon = pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i)$.

After production, each firm realizes the outcome of a state-invariant, exogenous exit shock. At that point, fraction $\chi \in (0,1)$ of firms exit the economy with $k' = b' = 0$. Each exiting firm undertakes a negative investment of size $(1-\delta)k$ and returns its cash as dividends to domestic households as it departs. Continuing incumbent firms choose their future capital stocks and debt $(b' > 0)$ or financial savings $(b' \leq 0)$ subject to a collateral constraint and non-negative dividends. Exiting firms are replaced at the start of the next period by an equal number of new firms that each begin with capital stock k_{0c}, debt b_{0c} and a productivity drawn from the ergodic distribution of ε; thus the total investment in new firms each period is χk_{0c}.

3.4.1 Static choices, profits and cash

Given its capital and productivity, the domestic real wage, real materials cost and relative price of its output, each firm chooses its labor and materials inputs to solve the following static problem, subject to the production technology in (20).

$$\max_{n,m} \omega_c(A)y(n,m;k,\varepsilon) - w_c(A)n - \frac{P_c^M(A)}{P_c(A)}m \tag{21}$$

The firm's labor and materials decision rules, and thus its output supply, follow immediately. That in turn recovers its static profits, $\pi_c(k,\varepsilon;A) = (1-\nu-\gamma)\omega_c y_c(k,\varepsilon;A)$, the real value of sales less flow input costs. Note that each of these is independent of its financial position.

$$n_c(k,\varepsilon;A) = \left[\gamma z_c \varepsilon \omega_c \left(\frac{\nu}{\gamma w_c}\right)^{1-\gamma} \left(\frac{P_c^M}{P_c}\right)^{-\gamma} k^\alpha\right]^{\frac{1}{1-\nu-\gamma}} \tag{22}$$

$$m_c(k,\varepsilon;A) = \gamma w_c \left(\nu \frac{P_c^M}{P_c}\right)^{-1} n_c(k,\varepsilon;A) \tag{23}$$

Given its static decision rules in (22) - (23), we can write the firm's profits as a multiplicative function of individual versus aggregate states in (24). This will be useful for determining its capital decision rule below.

$$\pi_c(k,\varepsilon;A) = (1-\nu-\gamma)\left[\varepsilon k^\alpha\right]^{\frac{1}{1-\nu-\gamma}} \Omega_c(A) \tag{24}$$

$$\text{where } \Omega_c(A) \equiv \left[z_c \omega_c(A)\gamma^\gamma \left(\frac{P_c^M(A)}{P_c(A)}\right)^{-\gamma} \left(\frac{\nu}{w_c(A)}\right)^\nu\right]^{\frac{1}{1-\nu-\gamma}} \tag{25}$$

We define x as the (k,b,ε) firm's real cash-on-hand, the sum of its static profit and non-depreciated capital net of outstanding debt:

$$x_c(k,b,\varepsilon;A) \equiv \pi_c(k,\varepsilon;A) + (1-\delta)k - b. \tag{26}$$

Exiting firms return their x to households as they leave the economy; continuing firms use it to finance capital investment and/or dividends, solving the intertemporal problem below.

3.4.2 Intertemporal choices

A continuing firm receives $q_c(A)$ units of domestic final goods in the current period for each unit of debt it incurs; thus, a debt with face value b' delivers a real loan of size $q_c(A)b'$. Capital accumulation is one period time-to-build; $k' = (1-\delta)k + i$, where i is investment. The firm's choices of k', b' and current dividends, D, are thus restricted by the budget constraint:

$$x_c(k,b,\varepsilon;A) + q_c(A)b' \geq D + k'. \tag{27}$$

We assume the firm cannot issue new equity to finance its investment, $D \geq 0$, and the debt it takes on is limited in proportion to its existing capital by the collateral constraint: $b' \leq \zeta_c(A)k$, where $\zeta_c \geq 0$ is the exogenous state variable reflecting the current availability of credit in country c.

Continuing firms make decisions taking as given the equilibrium evolution of the endogenous aggregate state, $S' = \Gamma(S, Z)$, and the transition probabilities π^Z_{lm} governing the evolution of the exogenous aggregate state Z. There are no real frictions impeding capital adjustments, so we can use the profit function (24) to explicitly solve for *target capitals* $k^*_c(\varepsilon; A)$ selected in absence of financial constraints. These ε-specific targets will facilitate our solution for firms' capital decisions.

We impose state-contingent discounting consistent with equilibrium in the market for shares (section 3.1) in stating firms' intertemporal problem. Let $\Lambda_c(A)$ be the value a country c firm ascribes to dividends, where $A = (S, Z)$, and assume firms discount their future value by the household discount factor β. In equilibrium, $\Lambda_c(A)$ is the domestic household marginal utility of consumption, so we are simply expressing the firm value function in units of marginal utility.

Let v^e_c represent the value of a country c firm just prior to the realization of the exit shock:

$$v^e_c(k, b, \varepsilon; A) = \chi \Lambda_c(A) x_c(k, b, \varepsilon; A) + (1 - \chi) v_c(k, b, \varepsilon; A), \tag{28}$$

where v_c is the expected discounted value conditional on continuing to the next period. Because the dividends of a continuing firm are immediate as a function of its k', b' choice from the binding budget constraint (27), we can write the problem of a continuing firm of type (k, b, ε_i) as follows.

$$\begin{aligned} v_c(k, b, \varepsilon_i; S, Z_l) = \max_{k', b'} \Big[& \Lambda_c(S, Z_l)[x_c(k, b, \varepsilon; S, Z_l) + q_c(S, Z_l)b' - k'] \\ & + \beta \sum_{m=1}^{N_Z} \sum_{j=1}^{N_\varepsilon} \pi^Z_{lm} \varphi^\varepsilon_{ij} v^e_c(k', b', \varepsilon_j; S', Z_m) \Big] \end{aligned} \tag{29}$$

$$\text{subject to:} \quad b' \leq \zeta_c(A)k \;\; \text{and} \;\; x_c(k, b, \varepsilon; A) + q_c(A)b' - k' \geq 0 \tag{30}$$

The second constraint in (30) prevents the firm from using negative dividends to evade the first.

The problem (29) - (30) is simplified further by the following observations. In equilibrium, no continuing firm can strictly raise its value by paying positive dividends, since it borrows and lends at the same price its owners face, and $\Lambda_c(A) = \lambda_c(A)$. If a firm has amassed sufficient cash to prevent its investment ever again being hindered by inadequate funds, its shadow value of retained earnings matches the household valuation of dividends. Any such *impervious* firm invests efficiently, and it is indifferent about paying dividends. Conversely, for a firm lacking adequate cash to preclude the possibility that the collateral constraint (30) may bind in some future state, the per-unit valuation of retained earnings exceeds the domestic household's dividend valuation. Any such financially *exposed* firm sets $D = 0$ to maximize its value, and its investment may be affected by its cash. With this classification, it is straightforward to recover the firm decision rules below.

Capital decision rules: Because no *real* frictions impede capital adjustment, an impervious firm maximizes its value by adopting the efficient (target) capital $k_c^*(\varepsilon_i; S, Z_l)$ satisfying:

$$\Lambda_c(S, Z_l) = \beta \sum_{m=1}^{N_Z} \pi_{lm}^Z \Lambda_c(S', Z_m) \left[1 + \delta + \sum_{j=1}^{N_\varepsilon} \varphi_{ij}^\varepsilon \partial \pi_c(k', \varepsilon_j; S', Z_m)/\partial k' \right].$$

Efficient capital choices for $i = 1, \ldots, N_\varepsilon$ are listed in (31), given $\Omega_c(A)$ in (25) and $S' = \Gamma(A)$, and the impervious firm capital rule is $k_c^*(\varepsilon_i; A)$.

$$k_c^*(\varepsilon_i; S, Z_l) = \left[\frac{\alpha}{1 - (1-\delta) q_c(S, Z_l)} \sum_{m=1}^{N_Z} \pi_{lm}^Z \frac{\beta \Lambda_c(S', Z_m)}{\Lambda_c(S, Z_l)} \Omega_c(S', Z_m) \sum_{j=1}^{N_\varepsilon} \varphi_{ij}^\varepsilon \varepsilon_j^{\frac{1}{1-\nu-\gamma}} \right]^{\frac{1-\nu-\gamma}{1-\alpha-\nu-\gamma}} \tag{31}$$

An exposed firm also maximizes its value by adopting $k_c^*(\varepsilon_i; A)$ *if* its cash permits. The collateral constraint (30) limits its b' and so bounds its affordable capital at $x_c(k, b, \varepsilon; A) + q_c(A)\zeta_c(A)k$. Given the strictly concave profit function, it adopts $k_c^*(\varepsilon; A)$ or the nearest k' not violating this bound. Thus, its capital rule is $g_c(k, b, \varepsilon; A) = \min\{k_c^*(\varepsilon; A), x_c(k, b, \varepsilon; A) + q_c(A)\zeta_c(A)k\}$. Note that g_c also captures an impervious firm's capital rule; its accumulated cash simply ensures a slack bound.

Debt decision rules: An *exposed* firm always sets $D = 0$, so its capital choice implies its debt through the binding budget constraint (27). Thus, it adopts $b' = [g_c(k, b, \varepsilon; A) - x_c(k, b, \varepsilon; A)]/q_c(A)$. By contrast, an impervious firm is financially indifferent, and we can choose how to resolve that indeterminacy so long as we ensure it remains so. One natural approach is to follow Khan and Thomas (2013) in assigning any such firm a *minimum savings* debt policy $B_c(\varepsilon_i; S, Z_l)$ ensuring it will have sufficient funds to invest efficiently without borrowing more than (30) allows in all possible future dates and states, and then retrieve its dividends from the budget constraint. In dynamic stochastic general equilibrium, $B_c(\varepsilon_i; S, Z_l)$ solves (32) - (33), where $\widetilde{B}_c\left(k_c^*(\varepsilon_i; S, Z_l), \varepsilon_j; S', Z_m\right)$ is the greatest debt a firm can hold, alongside capital $k_c^*(\varepsilon_i; S, Z_l)$, and still be impervious if next period's exogenous state is (ε_j, Z_m). The minimum $\widetilde{B}_c(\cdot)$ over all possible (ε_j, Z_m) is the greatest debt with which the firm can exit *this* period and know it will be impervious next period, $B_c(\varepsilon_i; A)$.

$$B_c(\varepsilon_i; S, Z_l) = \min_{\{\varepsilon_j | \varphi_{ij}^\varepsilon > 0 \text{ and } Z_m | \pi_{lm}^Z > 0\}} \widetilde{B}_c\Big(k_c^*(\varepsilon_i; S, Z_l), \varepsilon_j; \Gamma(S, Z_l), Z_m\Big) \tag{32}$$

$$\widetilde{B}_c(k, \varepsilon; A) = \pi_c(k, \varepsilon; A) + (1-\delta)k - k_c^*(\varepsilon; A) + q_c(A) \min\{B_c(\varepsilon; A), \zeta_c(A)k\} \tag{33}$$

Equation 33 identifies the maximum debt a firm can repay while adopting its target capital and a debt not exceeding that identified by the minimum savings policy; this is the b implying $D = 0$ when impervious firm rules are adopted. The minimum operator imposes the collateral constraint while identifying a firm borrowing $\zeta_c(A)k < B_c^I(\varepsilon; A)$ as impervious if it has sufficient cash to finance its investment. Any firm that can adopt $k' = k_c^*(\varepsilon; A)$ and $b' = \min\{B_c(\varepsilon; A), \zeta_c(A)k\}$ while paying non-negative dividends, $x_c(k, b, \varepsilon; A) - k_c^*(\varepsilon; A) + q_c(A)\min\{B_c(\varepsilon; A), \zeta_c(A)k\} \geq 0$, is impervious and remains so forever by adopting these policies.

Given our model's large aggregate state vector and number of relative price functions, we study its transitional dynamics following aggregate shocks rather than the full stochastic solution. This imposes ignorance of aggregate shocks in steady state, artificially restricting the information set for the minimum savings policy. We can ensure this alters no real results by recovering the minimum savings policy $B_c(\varepsilon_i; S_t, Z_t|Z_{t+1})$ date-by-date over the transition presuming knowledge of future exogenous aggregate states, and then verifying that all firms identified as impervious in the steady state can adopt their target capitals in every date of the impulse. Alternatively, we can assign impervious firms a zero-dividend policy $D_c^Z(k, b, \varepsilon; A) = 0$ ensuring they never regret their debt decisions. We adopt the second approach here, taking care to allow a sufficiently negative lower bound for the distribution of debt so that the fraction of firms at that lower bound never exceeds 0.1 percent and those there never find themselves unable to adopt their target capitals. One advantage of this is that we need not explicitly track which firms are impervious, since every firm's capital rule can be written as $g_c(k, b, \varepsilon; A) = \min\{k_c^*(\varepsilon; A), x_c(k, b, \varepsilon; A) + q_c(A)\zeta_c(A)k\}$, and every firm's debt rule is $f_c(k, b, \varepsilon; A) = [g_c(k, b, \varepsilon; A) - x_c(k, b, \varepsilon; A)]/q_c(A)$.

3.5 Recursive equilibrium

A *recursive competitive equilibrium* is a set of functions:

$$\varrho, Q, \{w_c, q_c, \rho_c, \widetilde{\rho}_c, P_c\}_{c=1,2},\ \{P_h^{Dc}, P_h^{X\widetilde{c}}, P_m^{Dc}, P_m^{X\widetilde{c}}, \omega_c\}_{c=1,2},\ \{V_c^h, C_c, N_c, \xi_c', \phi_c', B_c'\}_{c=1,2},$$
$$\{H_c, h^{Dc}, h^{X\widetilde{c}}, M_c, m^{Dc}, m^{X\widetilde{c}}\}_{c=1,2},\ \{W_{hc}, W_{mc}, Y_c, \}_{c=1,2}, \text{ and } \{n_c, m_c, y_c, g_c, f_c\}_{c=1,2}$$

that solve household, retailer and firm problems and clear the markets for assets, materials, final goods, labor, and firms' output in each country, as outlined by the following conditions.

(i) V_1^h solves (1) and V_2^h solves (6), with associated policy functions $(C_c, N_c, \xi_c', \phi_c', B_c')$ for $c = 1, 2$

(ii) final goods retailers solve (10), with policy functions $(H_c, h^{Dc}, h^{X\widetilde{c}})$ for $c = 1, 2$

(iii) material goods retailers solve (14), with policy functions $(M_c, m^{Dc}, m^{X\widetilde{c}})$ for $c = 1, 2$

(iv) wholesalers solve (18), with policy functions (W_{hc}, W_{mc}, Y_c) for $c = 1, 2$

(v) firms (k, b, ε) solve (21) with policy functions (n_c, m_c, y_c) for $c = 1, 2$

(vi) continuing firms solve (28) - (30) with associated policy functions (g_c, f_c), for $c = 1, 2$

(vii) $\xi_c'(k', b', \varepsilon_j, \xi_c, \phi_c, B_c; A) = \mu_c'(k', b', \varepsilon_j; A)$, for each $(k', b', \varepsilon_j) \in \mathcal{S}$ in country $c = 1, 2$

(viii) $\phi_c'(\xi_c, \phi_c, B_c; A) = \int f_c\,(k, b, \varepsilon; A)\,\mu_c(d\,[k \times b \times \varepsilon])$, for $c = 1, 2$

(ix) $\Psi_1 B_1'(A', \xi_1, \phi_1, B_1; A) + \Psi_2 B_2'(A', \xi_2, \phi_2, B_2; A) = 0$ for all $(A'; A)$

(x) $\Psi_c C_c(\xi_c, \phi_c, B_c; A) = \Psi_c H_c(A) - \Psi_c I_c(A)$, where:

$$\Psi_c I_c(A) \equiv \int [(1-\chi) g_c(k, b, \varepsilon; A) + \chi k_{0c} - (1-\delta)k]\mu_c(d\,[k \times b \times \varepsilon]), \text{ for } c = 1, 2$$

(xi) $\Psi_c N_c(\xi_c, \phi_c, B_c; A) = \int n_c(k, \varepsilon; A)\mu_c(d\,[k \times b \times \varepsilon])$, for $c = 1, 2$

(xii) $\Psi_c M_c(A) = \int m_c(k, \varepsilon; A)\mu_c(d\,[k \times b \times \varepsilon])$, for $c = 1, 2$

(xiii) $\Psi_c W_{hc}(A) = \Psi_c h^{Dc}(A) + \Psi_{\widetilde{c}} h^{X\widetilde{c}}(A)$, for $c = 1, 2$

(xiv) $\Psi_c W_{mc}(A) = \Psi_c m^{Dc}(A) + \Psi_{\widetilde{c}} m^{X\widetilde{c}}(A)$, for $c = 1, 2$

(xv) $\Psi_c\,[W_{hc}(A) + W_{mc}(A)] = \Psi_c Y_c(A)$, where:

$$\Psi_c Y_c(A) = \int y_c(k, b, \varepsilon; A)\mu_c(d\,[k \times b \times \varepsilon]), \text{ for } c = 1, 2$$

(xvi) $\mu_c'(\Upsilon, \varepsilon_j) = (1-\chi) \int_{\{(k,b,\varepsilon_i)\ |\ (g_c(k,b,\varepsilon_i;A), b_c'(k,b,\varepsilon_i;A)) \in \Upsilon\}} \varphi_{ij}^{\varepsilon} \mu_c(d\,[k \times b \times \varepsilon_i]) + \chi \mathcal{J}(k_0) M(\varepsilon_j)$,

$$\forall\,(\Upsilon, \varepsilon_j) \in \mathcal{S}, \text{defines } \Gamma, \text{ where } \mathcal{J}(k_0) = \{1 \text{ if } (k_0, 0) \in \Upsilon \text{ and } 0 \text{ otherwise}\}, \text{ for } c = 1, 2$$

GDP: We end this section with each country's real per-capita GDP in units of its own final goods, starting from the expenditure side: $GDP_c = C_c + I_c + NX_c$. Using items (x) and (xv) above with the NX_c equation (17) from section 3.2.3, price restrictions from section 3.3 and conditional factor demands from section 3.2, we can write this as a relative price weighted function of total production less material inputs: $GDP_c = \omega_c Y_c - \frac{P_c^M}{P_c} M_c$. This collapses to $GDP_c = (1-\gamma)\omega_c Y_c$ after aggregating firms' static first order conditions in section 3.4.1.

4 Calibration

The length of a period in our model is one quarter. We normalize country 1's population to 1 and calibrate its parameters using annualized moments drawn from postwar U.S. data. The only parameters differing in country 2 are those associated with its size and trade shares. We select these to reflect Canada's relative size and trading shares with the United States, as described below.[13]

We assume households have period utility: $u(C,N) = \frac{1}{1-\phi}\left(\left[C - \frac{\kappa}{\eta}N^{\eta}\right]^{1-\phi} - 1\right)$, adopting the preferences of Greenwood, Hercowitz and Huffman (1988). This specification is widely used in international business cycle models because it removes consumption from the intra-temporal marginal rate of substitution, thereby eliminating wealth effects on labor supply.[14] We set the household discount factor β to obtain a long-run annual real interest rate of 4 percent consistent with measurement in Gomme, Ravikumar and Rupert (2011). Next, we fix $\eta = 1.25$ to imply a labor supply elasticity of 4, and set the coefficient of relative risk aversion at $\phi = 2$ as in Backus, Kehoe and Kydland (1994), Kehoe and Perri (2002), Alessandria and Choi (2007) and elsewhere.

The capital depreciation rate δ is set to yield an average annual aggregate investment-to-capital ratio at 7.1 percent, consistent with that for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables, controlling for growth. We choose χ so that 9 percent of firms exit each year, guided by the 8.76 percent average among firms in the Bureau of Labor Statistics' Business Dynamics Statistics database (BDS) over 1980 - 2006, and we fix the materials share parameter γ at 0.43 to match the average materials share of manufacturing in the U.S. input-output tables from the World Input-Output Database over 2000-2014.[15] The remaining model parameters are jointly calibrated, though our discussion below will link parameters with the targets they most influence.

We set the capital share parameter α to reproduce the 2.34 average annual private capital-to-GDP ratio in the U.S. over 1954-2017, and the labor share parameter ν to imply a 60 percent labor share of GDP, as in Khan and Thomas (2013) and Perri and Quadrini (2018).[16] Next, we set the labor disutility parameter κ and country 1's steady-state exogenous productivity z_1^* so that its steady state GDP is 1, with households each working one-third of available time.

Country 2's relative population size Ψ_2 and steady-state exogenous TFP z_2^* are taken to ensure

[13] Our choice to pattern country 2 on Canada is prompted by its strong and relatively unhindered trade relationship with the U.S., as noted in section 1; we discuss other advantages of the choice in the next section.

[14] See, for example, Devereux, Gregory and Smith (1992), Schmitt-Grohé and Uribe (2003), and Alessandria, Kaboski and Midrigan (2013). Raffo (2008) shows its use in a standard two-country business cycle model helps to deliver the observed countercyclical net flow of goods across countries.

[15] Our average reflects all sectors other than agriculture and commodities and is similar to that for Canada, 0.48.

[16] A prior version of the model allowed a nontrivial constant returns-to-scale wholesale sector, with labor's share therein chosen to reproduce the U.S. average wholesale trade value added share of GDP, 5.97 percent. Given its size and frictionless design, we found the sector quantitatively irrelevant throughout our results.

its GDP averages 8.8 percent that in country 1, the average Canada-to-U.S. ratio over 1994-2007, while its per-capita labor hours match those in country 1. Turning to trade-related parameters, we set the elasticity of substitution between domestic and imported goods ρ at 0.9, as estimated by Heathcote and Perri (2002).[17] Next, we choose the four parameters reflecting home bias in each country's materials and final goods sectors, $\{\theta_{mc}, \theta_{hc}\}_{c=1,2}$, so that our model's steady state reproduces average U.S. imports from Canada relative to GDP (0.021) and Canadian imports from the U.S. relative to GDP (0.207) over 1994-2007 from the International Monetary Fund's Direction of Trade Statistics, alongside average U.S. materials inputs imported from Canada relative to total U.S. materials inputs use (0.017) and Canadian materials inputs imported from the U.S. relative to total Canadian materials inputs use (0.127) over 2000-2007.[18]

TABLE 1. Parameter values

(a) Population and preferences, retail sectors, mean exogenous aggregate tfp and credit states												
Ψ_2	β	ϕ	η	κ	ρ	θ_{h1}	θ_{m1}	θ_{h2}	θ_{m2}	z_1^*	z_2^*	ζ^*
0.070	0.99	2	1.25	2.367	0.9	0.995	0.989	0.907	0.896	2.119	2.856	0.505
(b) Production, death and birth, depreciation and idiosyncratic tfp shocks												
	α	ν	γ		χ	k_{01}	b_{01}	k_{02}	b_{02}	δ	ρ_ε	σ_ε
	0.164	0.342	0.43		0.023	1.451	0.580	1.447	0.579	0.018	0.901	0.029

NOTE.– Quarterly frequency; all parameter values rounded to nearest 0.001.

We turn now to idiosyncratic productivity, ordinary credit conditions, and new firms' initial states. Productivities are drawn from a log-normal distribution with persistence ρ_ε and standard deviation σ_ε, discretized using $N_\varepsilon = 7$ values. We set ρ_ε to imply a 0.659 annual persistence as in Khan and Thomas (2013), while σ_ε ensures that a comparable sample of mature firms in our model's steady state reproduces the 0.337 average cross-sectional standard deviation of annual investment rates reported by Cooper and Haltiwanger (2006) using panel data from the Longitudinal Research Database. Finally, we calibrate a common steady-state value for the collateral constraint anchor, ζ^*, and new firms' initial stocks $\{k_{0c}, b_{0c}\}_{c=1,2}$ so the debt-to-asset ratio of new firms in each country is 40 percent (Kauffman Firm Survey) and new and 1-year-old firms' average employment sizes relative to incumbent firms' match those in the BDS over 1990-2006, 27 and 37.1 percent, respectively. While not targeted in the calibration, our model's average employment share of young firms (aged 0 - 5 years) is 19.4 percent, versus 17.1 percent in the data.

[17] Corsetti et al. (2008) estimate a 0.85 elasticity of substitution between home and foreign tradeables using a symmetric two-country model calibrated to the U.S. and a trade-weighted aggregate of Canada, Japan and EU-15.

[18] The North American Free Trade Agreement took effect in 1994; hence our choice in computing average import shares. Our source for materials input shares, The World Input-Output Tables database, is available from 2000.

5 Results

We use our model to explore how a financial recession in a large, developed country like the United States affects its trading partners through international trade. We have selected Canada as our second country despite its small size, motivated by its proximity to the U.S. in both geographic and development terms, the fact that it is a leading trading partner for the U.S. and the fact that the U.S. is by far its most important trading partner. An advantage of this choice is that the great disparity in size implies minimal feedback effects from the small economy to the large one, allowing us an unobstructed look at the direct transmission of a U.S. credit shock to its neighbor.

The computational burden of solving stochastic simulations is great, given the two 3-dimensional distributions in our model's aggregate state vector and the large number of equilibrium pricing and forecasting functions involved. Thus, we study perfect-foresight transitions following unforeseeable aggregate shocks. While the approach is not without loss of generality, we have minimized its consequences for the misallocative effects of a credit tightening by imposing a zero-dividend rule for financially indifferent firms, ensuring no firm regrets financial decisions made in steady state.

We first examine responses when country 1 (U.S.) experiences a credit shock absent any shock in country 2 (Canada). Investigating the channel through which country 1's financial recession is transmitted abroad, we isolate the importance of trade balance adjustment in propagating the shock through investment reallocation across the two economies. Next, we highlight the aspects unique to this reallocative disturbance by contrasting our results to those following a country-1 productivity shock. Finally, we study a calibrated global credit crisis featuring financial shocks in both countries, there using a set of comparisons to disentangle how country 2's financial recession is reshaped, through trade, by the deep financial recession of its large trading partner.

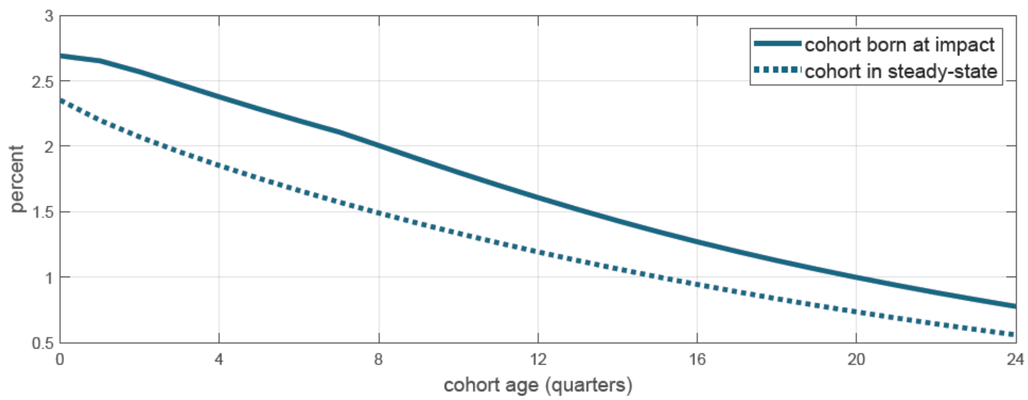
5.1 Transmission of a financial shock

We explore the repercussions of a country-1 credit shock by forcing the collateral constraint parameter ζ_1 below its steady state value, setting the shock's size and persistence so that the dynamic path of country-1 firms' aggregate debt resembles that of U.S. real commercial and industrial loans beginning in 2007. We fix ζ_1 at 69.29 percent of its steady-state value for 8 quarters, and then gradually return it to normal via an AR(1) process with persistence 0.935; this drives a 25.9 percent peak-to-trough decline in country-1 debt over 9 quarters and ensures the series is still 20.9 percent below its starting point one year later (in quarter 13), as observed in the United States.

All firms are subject to collateral constraints, but a domestic credit tightening sharply curtails some firms' investments while indirectly raising some others'. This uneven incidence distinguishes aggregate credit shocks from real ones, tilting the allocation of production further from the efficient one implied by firms' productivities.[19] The hardest hit firms are those most reliant on external funds, which tend to be the economy's youngest firms since entrants arrive with little capital and collateral constraints are backward-looking. Thus, disruptions to firms' life-cycle growth feature heavily in the paths of aggregate TFP and the return to domestic aggregate capital accumulation.

FIGURE 3: Life-cycle implications of a credit shock

(a) Average excess return to investment



(b) Average relative capital

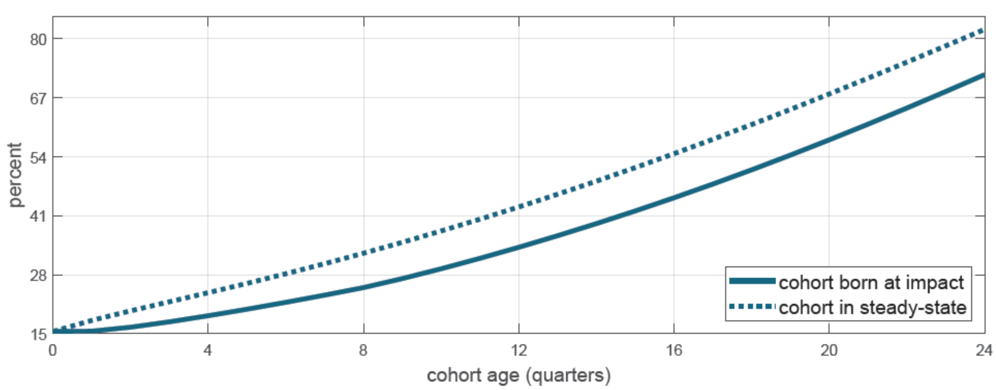
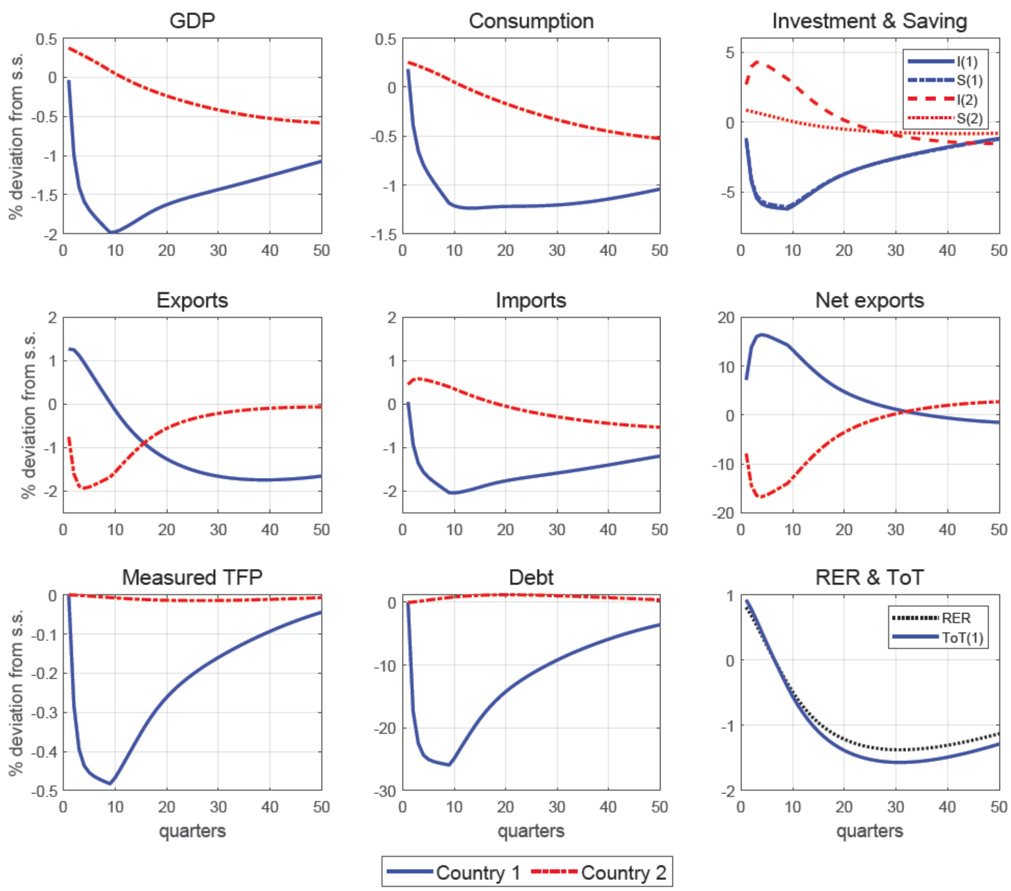


Figure 3 demonstrates this point more directly than did Khan and Thomas (2013), tracing how the shock alters life for a country-1 cohort arriving at its impact. Panel (a) plots the average excess return to investment (the gap between the expected discounted marginal value of adopted capital versus its unit purchase price) at each age, showing that firms born during a credit crisis have larger investment wedges than firms born in normal times. Though greatest in its earliest ages, this cohort's elevated inefficiency persists beyond its 8th year. Panel (b) shows its average relative

[19] Persistent idiosyncratic productivity shocks allow greater micro-level realism in our model, but are not essential to the misallocation channel described here. Figures establishing this are available on request.

capital (per-member average versus economywide average capital) is persistently lower than had it arrived in steady state, confirming that firms born during the crisis are more affected than those predating it. Together, the two panels establish that (a) young firms suffer a disproportionate share of the investment declines during a domestic financial recession, (b) their life-cycle growth phase is rendered unusually inefficient and hence protracted by it, and so (c) misallocation worsens with the birth of each new cohort for some time, as more cohorts are affected and the fraction of firms investing inefficiently grows. Following such persistent disruptions to the growth phases of cumulated cohorts, country 1's distribution of capital is slow to recover its usual shape, as is evident from the path of measured TFP in our next figure.

FIGURE 4. Global responses to a country-1 credit shock



NOTE.– Responses to an 8-quarter, 30.7 percent credit tightening in country 1 with AR(1) recovery.

Figure 4 presents both countries' aggregate responses to the country-1 credit shock. Recall that each country's GDP is a price-weighted function of its firms' total production; $GDP_c = (1-\gamma)\omega_c Y_c$, for $c = 1, 2$. Given growing distortions in its capital allocation implied by our discussion above, country 1's aggregate productivity erosion discourages investment and hours worked, compounding the shock's direct effects in lowering GDP.[20] These three series reach their troughs after 8 quarters as credit conditions begin improving; whereas, consumption falls and recovers with a slight lag, after initially rising in anticipation of lowered returns to saving.[21]

In a closed-economy model, any decline in country-1 investment would be met by an equal fall in its aggregate saving, $S = GDP - C$. In our setting, however, country-1 households effectively relocate some savings abroad in response to the negative wealth effect implied by their worsening recession. The lesser reduction in their saving relative to firms' total investment, or *residual saving*, is mirrored in a persistent improvement of the country-1 trade balance, with the reduced demand for imports initially reinforcing a rise in exports and thereafter offsetting its decline. This trade balance adjustment is only a small fraction of GDP for country 1, as it is almost a closed economy in our calibration. However, it has a non-trivial impact on country 2, where international trade accounts for a larger share of the aggregate economy.

While no direct shock hits country 2, country 1's credit recession induces an immediate rise in its GDP followed by a far longer contraction. A small consumption rise accounts for some part, but the early phase of this response is mostly driven by rises in investment. With tight credit worsening the allocation of capital in country 1, and thus its aggregate return, some investment relocates to country 2. This is facilitated by country-2 households' borrowing from international financial markets, as reflected in country 2's trade balance deterioration and negative residual saving.

Understanding their real cost of imports will steadily rise as country 1's misallocation problem grows, country 2's retailers prepare to start raising the domestic share of their productive input mix in coming dates. Large rises in investment put additional capital in place to help offset the persistently high domestic marginal costs this shift will imply. Those in turn are achieved by a temporary cheap-imports windfall alongside increased hours worked. With no rise in exogenous TFP to help fuel the expansion, there is no direct substitution effect reinforcing savings incentives. Instead, capital grows least among young, cash-poor firms that on average need it most, so TFP falls as the aggregate stock grows, thereby encouraging early-date rises in consumption.

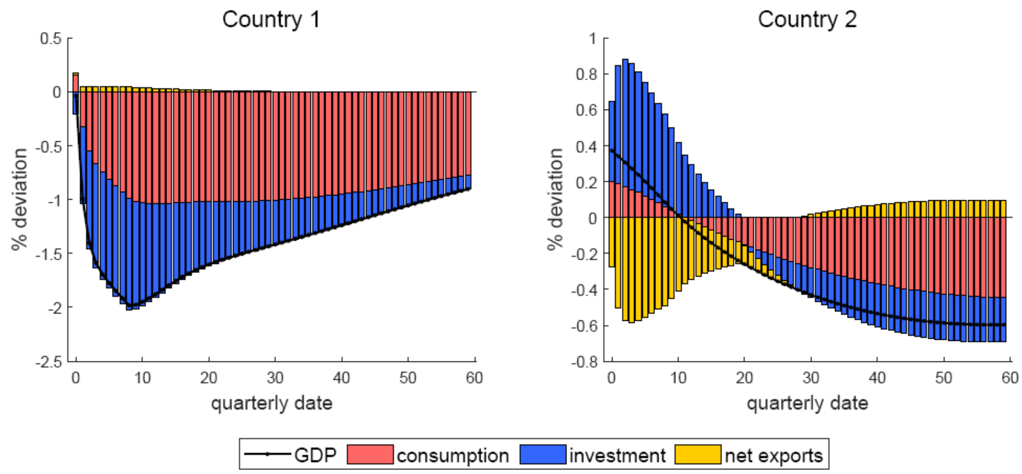
Over time, as the country-1 credit state gradually recovers, country 2's investment boom fades and its initial capital accumulation is reversed thereafter. While slow for reasons explained above,

[20] Our peak-to-trough declines in country 1 GDP and investment are smaller than in Khan and Thomas' (2013) closed economy study for three main reasons. First, country 1 new firms have capital stocks roughly 16 percent of the average stock in our calibration, whereas Khan and Thomas' entrants had 10 percent of the average stock and so were more reliant on debt and for longer. Second, our exit rate is 1 percentage point lower, implying fewer firms in the maturing phase most hurt by financial tightening. Third, our country 1 credit shock is smaller and shorter, with a 16 (versus 88) percentage point reduction in ζ_1 held in place for 2 (versus 4) years before mean reversion begins.

[21] Total hours worked responses closely resemble the paths of GDP in each country; we omit them for sake of space.

country 1's steady recovery in aggregate productivity starting in quarter 10 gradually shifts global production back in its direction. The demand for country 2's exports is persistently depressed in this episode, and ultimately so is its domestic demand for investment. There follows a long contraction in country 2 as 'excess' capital gradually relocates and the disruption to its own productive distribution is slowly repaired. GDP reaches its trough in quarter 58, and the subsequent recovery is gradual.

FIGURE 5. Country-1 credit shock: GDP response decompositions



If figure 4 suggested that investment is central to domestic *and* cross-country transmission of country 1's financial shock, figure 5 reinforces that message. Here, we decompose the two GDP responses to consider the contributions from each of their three expenditure-side components. Over the 8-quarter downturn in country 1, declines in investment account for on average 59 percent of its GDP losses, while net exports has almost no role given country 1's small trade share in our calibration. During the many subsequent quarters of the recovery phase, investment comes back fairly steadily while consumption increasingly offsets that recovery, holding GDP down.

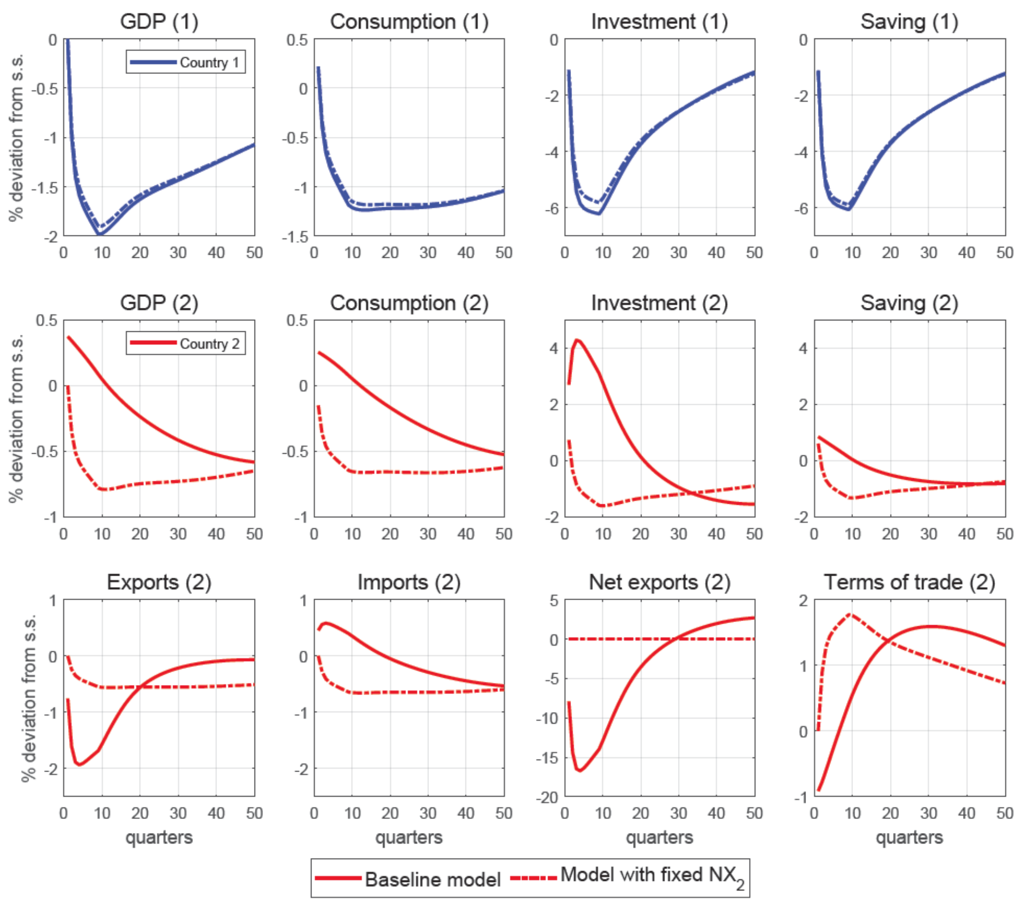
Looking rightward, we see investment also has an outsized role in transmitting the shock's effects to country 2. There, the initial rise in consumption and far greater rises in investment together more than offset the fall in net exports that accompanies the reduced demand for its exports we discussed above. When country 1 starts its recovery, country 2's long contraction begins. There onward, til its GDP trough 49 quarters later, investment has an increasingly prominent role in depressing economic activity and ultimately contributes more than 40 percent to the contraction.[22]

Figure 6 confirms the key role changes in the trade balance have in the transmission mechanics discussed above by studying what happens in their near-absence, fixing country 2's net exports

[22] The appendix demonstrates that all aspects emphasized in our discussion of figures 4 - 5 are stronger in a model specification where traded goods are not complements, but substitutes (section A.1) and robust to the removal of both cross-country population differences (section A.2) and input-output linkages in production (section A.3).

at its marginally positive steady-state level via changes in the real exchange rate. In essence, this exercise prevents additional international borrowing and lending in response to the country-1 financial shock.[23] The modification alters little for country 1 (row 1) beyond the underlying trade-related variables, as net exports are such a small fraction of its GDP, though investment naturally falls slightly less than in the baseline (solid line) response.

FIGURE 6. Country-1 credit shock: Role of net export changes



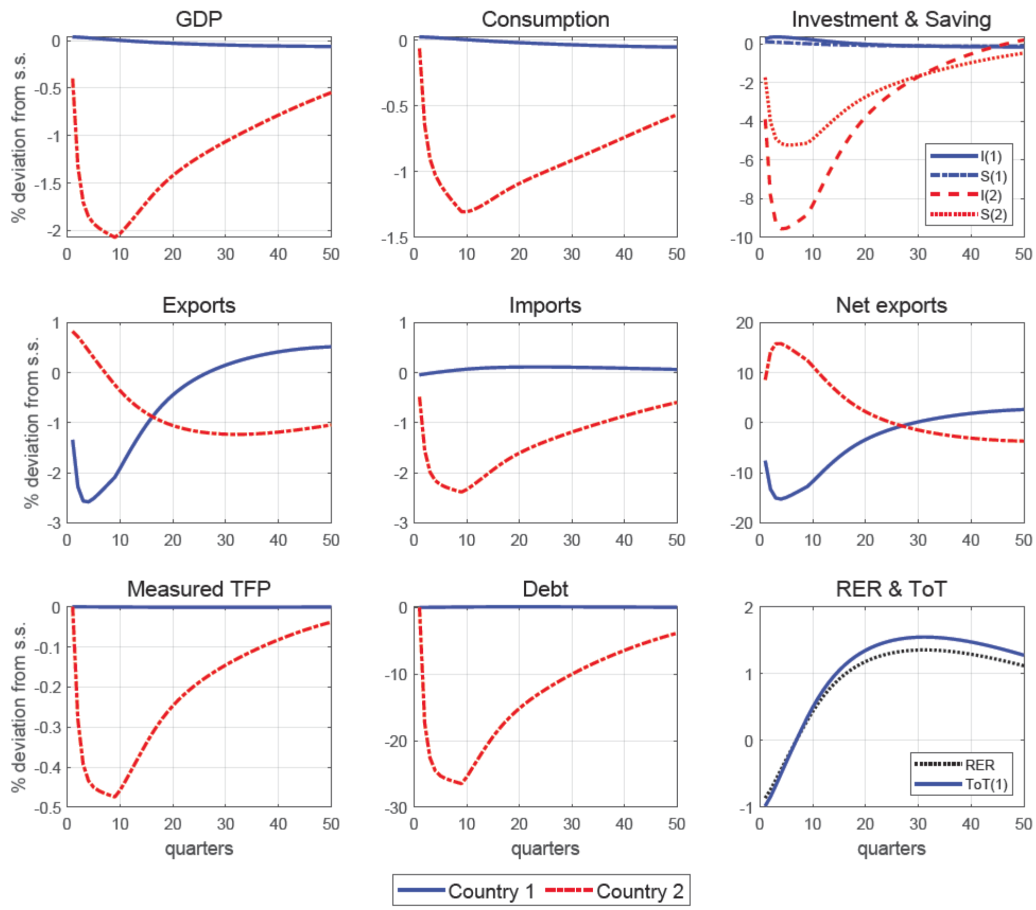
By contrast, country 2's responses (rows 2 and 3) are substantially altered when the sizeable per-capita net inflow of country-1 goods is suppressed. In this case, the decline in country-2 exports must be met by an equal fall in its imports, unlike the (solid line) rise of the baseline response, and its terms of trade immediately begin depreciating. Raised investment in preparation for the coming long episode of high import prices is sharply curtailed, at just 0.73 percent above normal, and it lasts only one quarter. With the investment boom eliminated by prohibiting its households raising additional funds from international capital markets, country 2's contraction begins immediately and is deeper than in the baseline model. Though the recovery starts 47 quarters sooner, shortly

[23] We thank an anonymous referee for suggesting this exercise linking the current and capital account adjustments.

after that in country 1, its half-life is 89 quarters (versus the baseline 75).

Throughout the discussion above, we have focused on the transmission of a financial shock from a large economy to a small open economy. We close this subsection by emphasizing that the mechanics of that transmission do not apply in reverse when the smaller economy instead directly experiences the shock. Figure 7 presents our model's responses to a country-2 credit shock of exactly the same size and persistence as country 1 experienced in figure 4. It shows the asymmetries between these economies matter enormously for the cross-country transmission of an aggregate shock.[24]

FIGURE 7. Global responses to a country-2 credit shock



Country 2's financial recession looks much like country 1's in figure 4, with the same domestic transmission playing out. Despite similar (reversed) net export dynamics, the implications of these changes for country 1 bear little resemblance to country 2's responses in our earlier figure. Recall that country 2's GDP is 8.8 percent that of country 1, and imports from country 2 amount to only 2.1 percent of country 1's GDP, given our U.S. - Canada calibration. As such, the rise in country-2

[24] This is also evident in our model's responses to TFP shocks; figures available on request. Appendix section A2 shows that the key factor influencing transmission is the overall share of the external sector in the recipient economy.

exports largely reflects real exchange rate movement and shows up imperceptibly in country 1's imports. Given its limited exposure to trade with country 2, country 1's GDP, consumption and investment are almost entirely unaffected by the country-2 credit shock.

5.2 Transmission of a real shock

Our preceding analysis of the transmission of financial shocks isolated investment's central role in propagating a shock both domestically and abroad. This offers new insights to the international real business cycle literature, wherein the majority of analysis has been confined to real shocks such as aggregate productivity shocks. For sake of comparison, we next examine the transmission of a negative TFP shock in our model, considering how its channels relate to those emphasized above.

FIGURE 8. Global responses to a country-1 productivity shock

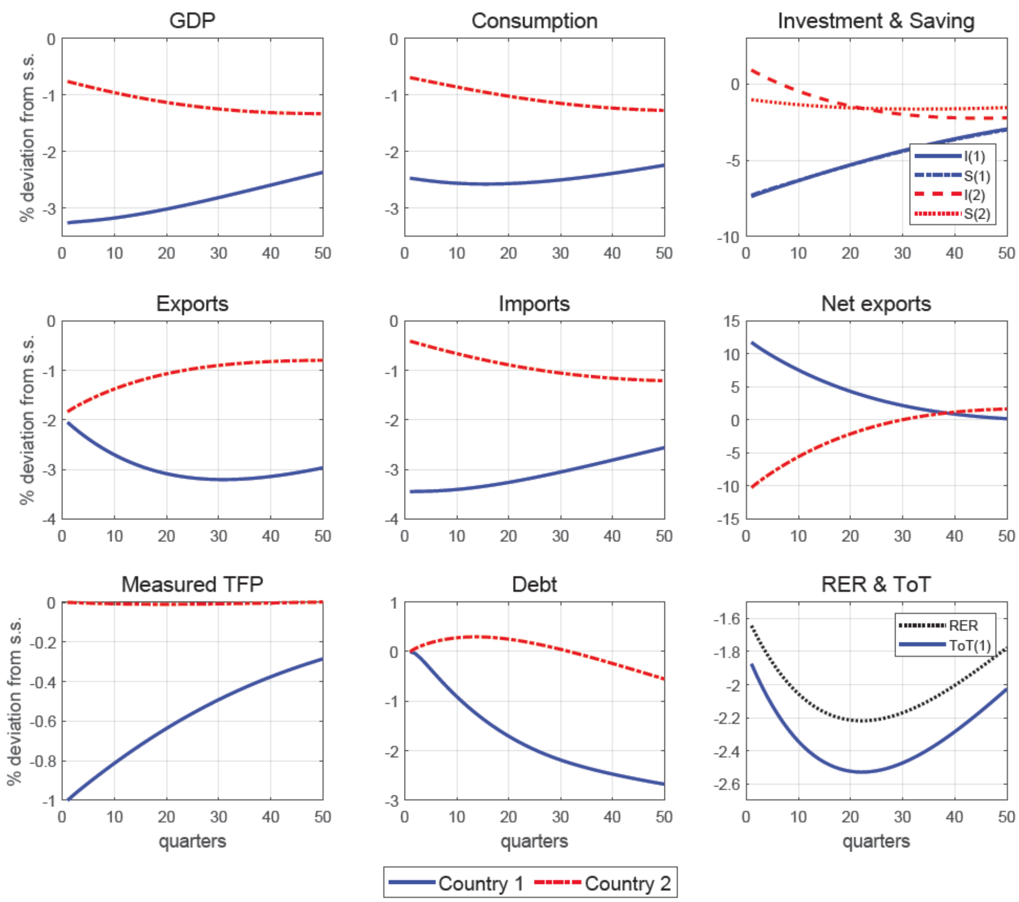
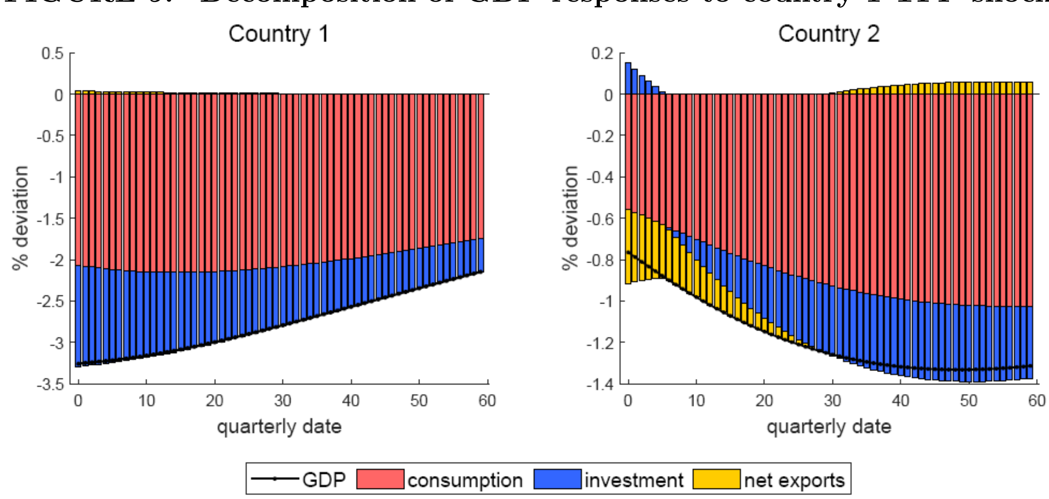


Figure 8 presents global responses to a 1-percent productivity shock in country 1 followed by AR(1) recovery with persistence 0.977. Unlike the domestic results of the financial shock in figures 3 and 4, where tight credit amplified misallocation by disproportionately depressing young, growing firms' investments, the TFP shock scales country-1 firms' investments down evenly in proportion to their usual productive shares. This mirrors Khan and Thomas' (2013) closed-economy finding; with no change in the allocation of production, the aggregate productivity response coincides with the shock, so other domestic responses resemble those in a frictionless representative firm model.

Unlike the case of a country-1 financial shock, there are no cheap-imports windfalls to fuel an initial investment-led expansion in country 2 in response to this shock. Country 1's abrupt fall in productivity leads it to lower (not raise) its exports to country 2 and immediately (not gradually) lower its imports. Meanwhile, its sharp drop in production worsens the terms of trade for country 2; by contrast, country 2's terms of trade initially appreciated in figure 4 to accommodate a sizeable inflow of goods. These events lead country 2 households to work less, absent wealth effects in their supply decisions, and GDP immediately contracts.[25] Investment rises temporarily (far less than in figure 4) to get a bit more capital in place towards shifting retailers' production mix in favor of domestic goods, but none of this rise is financed by country 1 households. Country 2 households soften the immediate fall in their consumption by lowering (not raising) their savings, so the forces underlying the fall in the current account differ from those following country 1's credit shock.

We close this subsection with the novel observation that the distinctions in how financial versus real shocks operate within a country extend to how these shocks affect its trading partners. Figure 9 decomposes each country's GDP response into its expenditure-side components following country 1's TFP shock for comparison with those in figure 5 following its credit shock. Whereas investment played a central role in propagating the financial shock domestically and abroad in figure 5, here, we see the majority of both GDP responses to the real shock can be attributed to consumption.

FIGURE 9. Decomposition of GDP responses to country-1 TFP shock



[25] Miyamoto and Nguyen (2017) analyze the transmission of a permanent productivity shock in an IRBC model similar to ours in its inclusion of imported intermediate inputs and weak wealth effects on labor supply.

Starting on the left with country 1, consumption losses contribute roughly two-thirds of the initial GDP decline, and that share grows with time. Meanwhile, country 2 investment rises for a few quarters in response to the immediate rise in its relative productivity. However, unlike the investment boom following country 1's credit shock in figure 5, these rises are modest, short-lived, and dwarfed by declines in consumption. Investment's share of GDP never moves far from its usual 16.6 percent, and the contraction in consumption is the dominant factor throughout the GDP response. In sum, our analysis reveals that a financial shock propagates through a very different channel in comparison with the real shock most commonly studied in the literature.

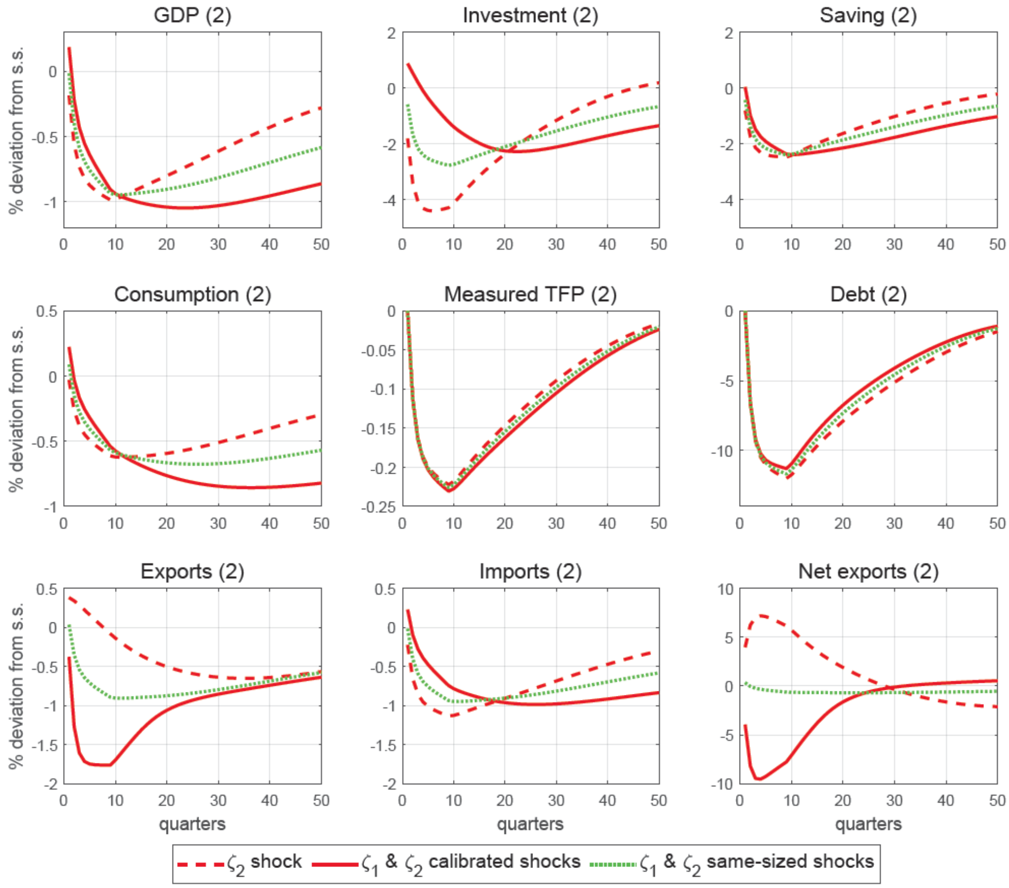
5.3 Global financial shocks

We have confined attention so far to the transmission of one aggregate shock, with emphasis on the consequences of a country-1 credit shock calibrated to the United States 2007-2009 experience. As discussed in section 1, however, there is ample evidence indicating that the U.S. financial crisis spilled into a global financial crisis affecting many advanced economies. The tightening of bank lending standards and subsequent fall in Canadian business loans suggests that Canada was among them, though its recession was less severe and shorter than in most other G7 countries.

Here, we allow for financial contagion by examining simultaneous credit shocks in our model counterparts to the U.S. and Canada, and highlight how international trade transmission alters the consequences of the domestic financial shock for Canada. Retaining the calibrated country-1 shock above, we drop country 2's collateral constraint parameter, ζ_2, to 86.18 percent of its steady-state value so that its peak-to-trough debt decline matches that of real loans to Canadian non-financial corporations and unincorporated businesses, at 11.4 percent. The credit recovery path starting in quarter 9 follows the same AR(1) process as in country 1, and implies that country 2's debt series is still 9.33 percent below normal one year later, as happened in Canada.

Figure 10 compares three sets of impulse responses to isolate how our small open economy's financial recession is altered through international trade by the fact that its large trading partner is experiencing a deep financial recession. The dashed series in each panel are country 2's responses to its domestic credit shock absent any country-1 shock, dotted series are its responses if country 1 experiences the same shock, while solid series are its responses when country 1 is hit by the calibrated ζ_1 shock. The gaps between the solid and dashed responses reflect the consequences of country 1's credit crisis transmitted to country 2 through trade, the channel explored in isolation above in figure 4; the gaps between the dotted and dashed responses show what that channel would have contributed had country 1's financial recession been less severe.

FIGURE 10. Country 2 responses to global financial shocks



NOTE.– Country 2 responses to an 8-quarter 13.8 percent credit shock accompanied by: no country-1 shock, the same shock in country 1, or the calibrated U.S. shock in country 1.

Considering the solid versus dashed series, we see that country 2's recession is altered in three notable ways by the deep recession in country 1. First, the early phase of the downturn is softened by an investment rise effectively financed by country-1 households, where investment otherwise should have fallen roughly 2 percent right away; for example, the GDP contraction in quarter 2 is roughly half what it would have been without the net inflow of goods reflected in country 2's trade balance. Although dampened by the presence of the ζ_1 shock, note that the ultimate fall in investment is still 12 times the ultimate 0.23 percent fall in aggregate productivity; whereas, country-2 investment falls 8.16 percent in response to a direct 1 percent productivity shock.

Second, given the countervailing force of the resulting stock accumulation, the recession lasts more than twice as long as it otherwise would have, with GDP reaching its trough in quarter 24 versus quarter 9. Third, and most strikingly, the recovery phase is greatly protracted by the slow relocation of 'excess' capital back to country 1, and by the fact that its arrival at a time of heightened financial distortions implies a greater distributional disruption to be repaired later.

Investment is still 1.5 percent below normal 11.5 years out, where it would have recovered fully absent the country 1 credit shock; meanwhile, measuring from their respective trough dates, the half-life of the GDP response is extended by more than a decade, from 7 to 17.5 years.

Turning to the dotted versus dashed responses, notice country 2's initial GDP downturn still would be moderated by country 1's recession if its source was the same credit shock as in country 2, if less so. However, the more modest country-1 recession would merely soften country 2's investment decline, rather than pushing investment above normal for several quarters, and so would not delay the GDP trough. Nonetheless, country 2's failure to repurpose as much capital as it otherwise should have still would extend the half-life of its recovery by 13 years. These observations suggest that the size of the country-1 financial recession is not so critical in reshaping the country-2 financial recession as is country 2's exposure to trade with its large neighbor.

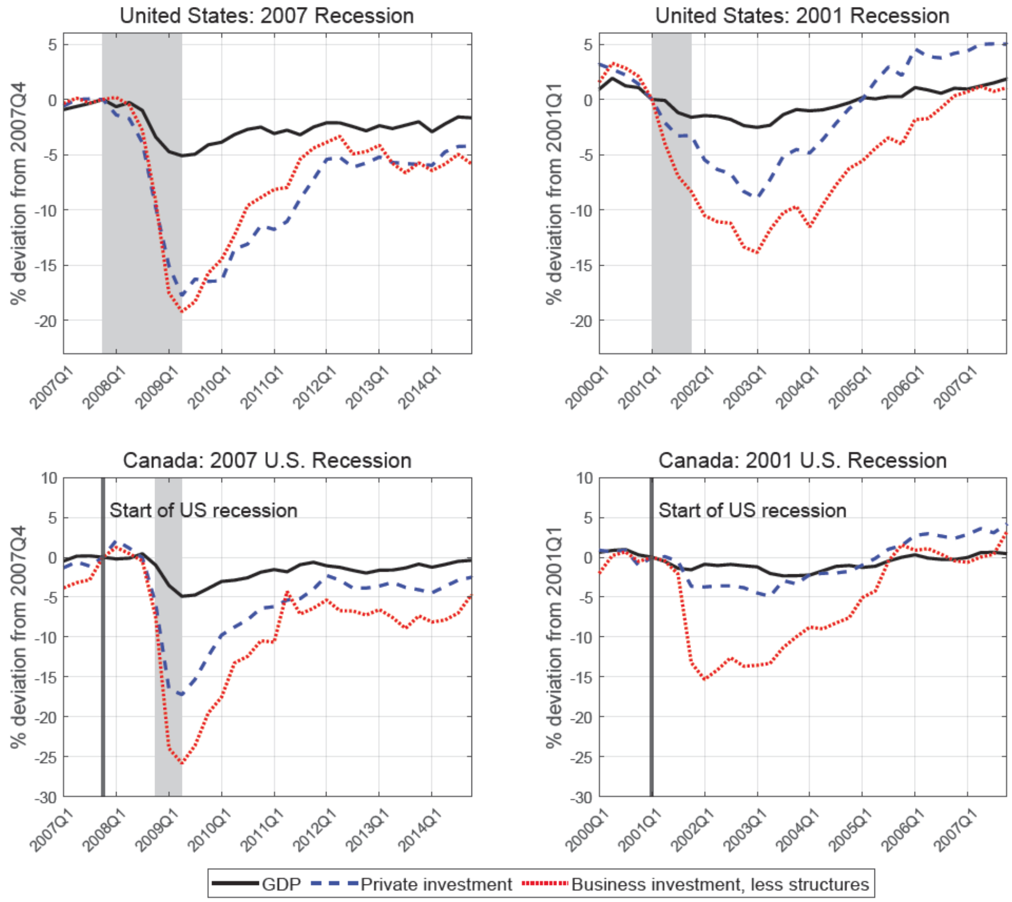
5.4 Connections to the U.S. - Canada experience

As discussed in section 1 and elsewhere in the literature, the onset of the U.S. financial crisis was soon followed by large, unusually synchronized, real contractions across many advanced and emerging market economies, with Canada among them. Taking as given that financial contagion was instrumental in driving the business cycle synchronization over this episode, our model's predictions in section 5.3 suggest that international goods trade also may have had a role in transmitting the effects of the U.S. shock to its trading partners, especially ones with comparatively large exposure to trade. Those results in conjunction with the analysis from section 5.1 indicate that the dynamics of investment and the changes in the underlying distributions of firms that prompt them are central in propagating financial market disturbances within and across countries.

Confronted with synchronized credit shocks, our model predicts lengthy investment contractions of considerable size when compared with the declines in aggregate productivity, and that these disproportionately affect young, growing firms. The latter fact has persistent implications for both countries' firm distributions, slowing aggregate investment and TFP reversion and thus recoveries in aggregate capital and GDP. Country 2's recovery is further gradualized by the extra stock and added distributional disruption implied by early date investment infusions from country-1 households. These results are broadly consistent with the prolonged stagnation of investment seen in the United States and Canada following the global financial crisis, to which we now turn.

Figure 11 presents log, HP-filtered real GDP, private investment, and business fixed investment less structures in the U.S. and Canada in dates surrounding the U.S. 2007 (left panels) and 2001 recessions, with each series demeaned to its value at the start of the corresponding U.S. recession.

FIGURE 11. Business cycle dynamics in the U.S. and Canada



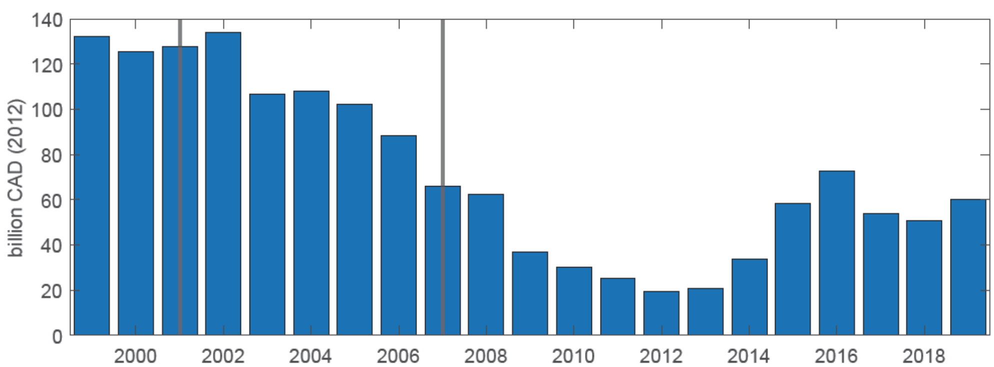
NOTE.– Real GDP, private investment and business fixed investment series are from the U.S. Bureau of Economic Analysis and Statistics Canada GDP tables. Private investment includes consumer durables and residential investment. All series are in logs, detrended using an HP filter with weight 1600. The U.S. filter is constructed using 1954Q1 - 2015Q2 data; the Canadian filter uses 1961Q1 - 2015Q2 data.

The Canadian recession beginning in 2008Q4 was less severe than those in many other advanced economies, as it had no banking crisis per se. Unlike the U.S. and many other countries, Canada did not experience bank failures or government bank bailouts during the global financial crisis, given tighter regulation of its financial sector; see Bordo, Redish and Rockoff (2011). It also experienced only about half the percentage decline in real C&I loans as happened in the United States, recalling our discussion in section 1. These observations notwithstanding, the size of its GDP contraction and that in the U.S. are a near perfect match. This is at odds with the predictions of our model, which translates a roughly half-sized fall in business lending into a roughly half-sized fall in GDP; however, some of the discrepancy might be explained by a real shock affecting Canada around this time that is omitted from our analysis, a notion we will return to below.

Canadian investment initially rose in early 2008. Starting in 2008Q3, private investment began a steep decline outstripped by the fall in business investment. Model counterparts in figure 10 qualitatively agree with these observations, though the predicted investment decline is far smaller and more gradual. Whereas the empirical investment trough dates coincided in the 2007 recession episode, our model reproduces that coincidence only if we reduce the size of the country-1 credit shock to that in country-2. Over the recovery, United States and Canadian investment came back fairly steadily between mid-2009 and mid-2011, but then stagnated over several years.[26]

Our calibrated financial shocks imply each country's credit variable is low for 2 years, then slowly returns to normal. Given these shock paths, our model's timing is perhaps not surprisingly off in predicting GDP trough dates well after those in the data. Its predictions of a considerably later trough date and far slower subsequent recovery in Canada than the U.S. are more problematic. It may be worth noting that the Canadian economy was buffeted by sharp declines in commodity prices and exports that began in mid-2008 and hit bottom in early 2009. Furthermore, monetary and fiscal stimulus packages helped hasten the actual economy's subsequent recovery. These factors absent from our analysis presumably contribute to the differences in timing.

FIGURE 12. Canada's real trade balance with the U.S.



NOTE.– Real trade balance for goods excluding energy products. Nominal exports and imports are deflated using export and import price indices. Data source: Statistics Canada.

Our model's key trade-related prediction underlying the strong influence of country 1's recession on that in country 2 is a large, persistent fall in country 2's trade balance. Figure 12 shows Canada's real surplus in non-energy goods trade with the U.S. fell during the global crisis. While in decline since the early 2000s, reductions in this series starting in 2008 were more pronounced and persisted

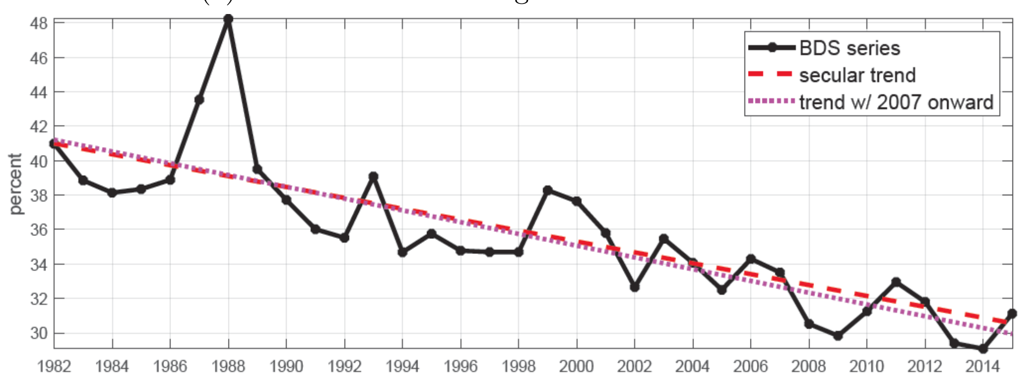
[26] Note the contrast between these observations and the investment dynamics surrounding the U.S. 2001 recession. While both countries' detrended private and business investment series fell well below their 2001Q1 levels, the declines were small in comparison with those in the left panel, they were more gradual, and the subsequent recoveries in these series were more sustained. Note also that Canada experienced no recession in the 2001 episode.

for several years. These observations are consistent with predictions in figure 10. At the same time, we must acknowledge that the country 2 export decline there is far smaller than its empirical counterpart in figure 2, and the coincident rise in country 1's exports is at odds with that figure.

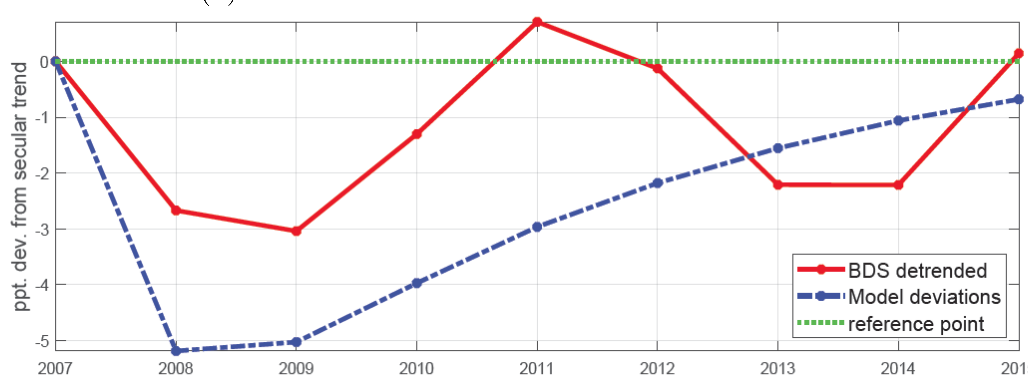
We began this section by demonstrating that disruptions to young firms' life-cycle growth are central to credit-shock transmission in our model, so we close the discussion by considering how this prediction lines up with the 2007 recession episode. Absent publicly available data on Canada's age-size distribution, we compare the model's country-1 predictions with U.S. firm data from the Business Dynamics Statistics database. Annualizing our model-generated series for comparison with the BDS, we focus on the average relative size of age-1 firms at each year, since this series best reflects disproportionately increased inefficiencies in new firms' investments from the prior year.

FIGURE 13. Average relative size of 1-year-old firms

(a) Secular decline among BDS firms over 1982-2015



(b) Deviations from trend: BDS data versus model



The top panel of figure 13 plots age-1 U.S. firms' average relative size over 1982 - 2015 against a (dashed red) linearly-extrapolated trend based on observations over 1982 - 2006. This line serves as our reference in constructing empirical deviations from trend; the second trend is there only to show how subsequent events hastened decline in the series. The detrended series is demeaned to its 2007 value and plotted against model deviations from steady-state in the bottom panel. Admittedly, the relative employment contraction suffered by the model's age-1 firms is greater and more persistent than what their counterparts in the BDS experienced. However, the paths of these series broadly agree from 2007 through 2011. Both show a sizeable initial decline lasting two years and, to differing degrees, gradualism in the recoveries of age-1 firms to their usual productive shares.

6 Concluding remarks

We have developed a two-country equilibrium business cycle model to study the international transmission of financial shocks through bilateral trade. Our model's persistent heterogeneity in firms' needs for and access to credit is central to its misallocation-driven domestic propagation of an aggregate credit tightening. Its inclusion of input-output linkages within and across countries accommodates an empirically valid role for trade in intermediate inputs. Finally, its calibration to the U.S. and Canada reflects an epicenter of the global financial crisis interacting with one of its leading trade partners, but at the same time reflects the interactions of a large economy relatively impervious to trade with a smaller economy highly reliant on its external sector.

Examining the model's responses following a financial shock calibrated to the path of U.S. business loans in the 2007 crisis, we have seen that the unique mechanics translating aggregate credit disruptions into real ones in closed-economy settings extend across borders. Tight credit conditions in the large economy are most destructive to young firms' life-cycle growth, and so drive persistently worsened misallocation and lowered expected returns to domestic aggregate capital accumulation with the arrival of each new cohort. This explains investment's far larger share in the decomposition of a credit-induced GDP contraction compared with that induced by an aggregate TFP shock, and why the same pattern emerges for the trading partner experiencing no direct shock.

A persistent trade balance adjustment facilitates shifts in the composition of world production whilst returns to investment are falling in the large economy. Local households reduce their savings by less than the fall in domestic investment and effectively invest the residual savings abroad through raised net exports. The resulting inflows are substantial relative to the small economy's usual productive scale, and help finance large rises in its investment to expand domestic production in anticipation of rising import prices. Over time, as credit conditions improve and the large economy's capital distribution slowly recovers its normal shape, world production shifts back in that direction, prompting persistent reversals in both countries' investment series and a prolonged GDP contraction in the small economy. Comparing these dynamics to the coincident consumption-led recessions in both countries following an aggregate TFP shock in the large economy, our results indicate that the dominant trade channel propagating an aggregate shock internationally is principally reflective of its domestic transmission channel.

Confronting the small economy with a credit shock reproducing the fall in Canadian business loans over the global financial crisis, we have seen that its recession is of little consequence to its far larger, comparatively closed, trading partner. Conversely, through the mechanics reviewed above, the deep financial recession abroad reshapes the small economy's recession, delaying the start of its recovery phase and greatly extending it. Taking financial contagion as given, this suggests that international trade had a distinct, non-negligible role in the persistent stagnation of investment and slow economic recoveries following the global financial crisis. Some results from this exercise align well with U.S. - Canada evidence; those that do not indicate that a full Canadian calibration of the second economy and stochastic simulation with jointly estimated financial and real shocks would be necessary to robustly confirm this prediction. We leave that investigation for future work.

Our analysis also might be extended to explore recent findings on the disaggregated sources of the international trade collapse coinciding with the global financial crisis. Levchenko, Lewis and Tesar (2010) and Eaton, Kortum, Neiman and Romalis (2016) argue that the collapse came largely from declines in expenditures on durable goods, which are trade-intensive. In fact, the business cycle accounting decomposition undertaken by Eaton et al. (2016) attributes two-thirds of the decline in trade relative to GDP during that period to shocks affecting the efficiency of investment in durable manufactures. Because investment dynamics play the central role in propagating financial shocks in our model economy, a carefully calibrated extension including durable and nondurable goods sectors could be useful in tracing out the mechanisms underlying their result, thereby shedding further light on real-financial linkages through international trade.

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A Appendix

This appendix considers how our model's predictions are affected by three key assumptions made in its development and calibration. We focus on our baseline responses to the country-1 financial shock analyzed in section 5.1 for clarity sake, and consider how those results change if we remove: (i) complementarity between domestic and foreign goods, (ii) differences in country sizes and trade shares, or (iii) input-output linkages in production.

A.1 Complementarity

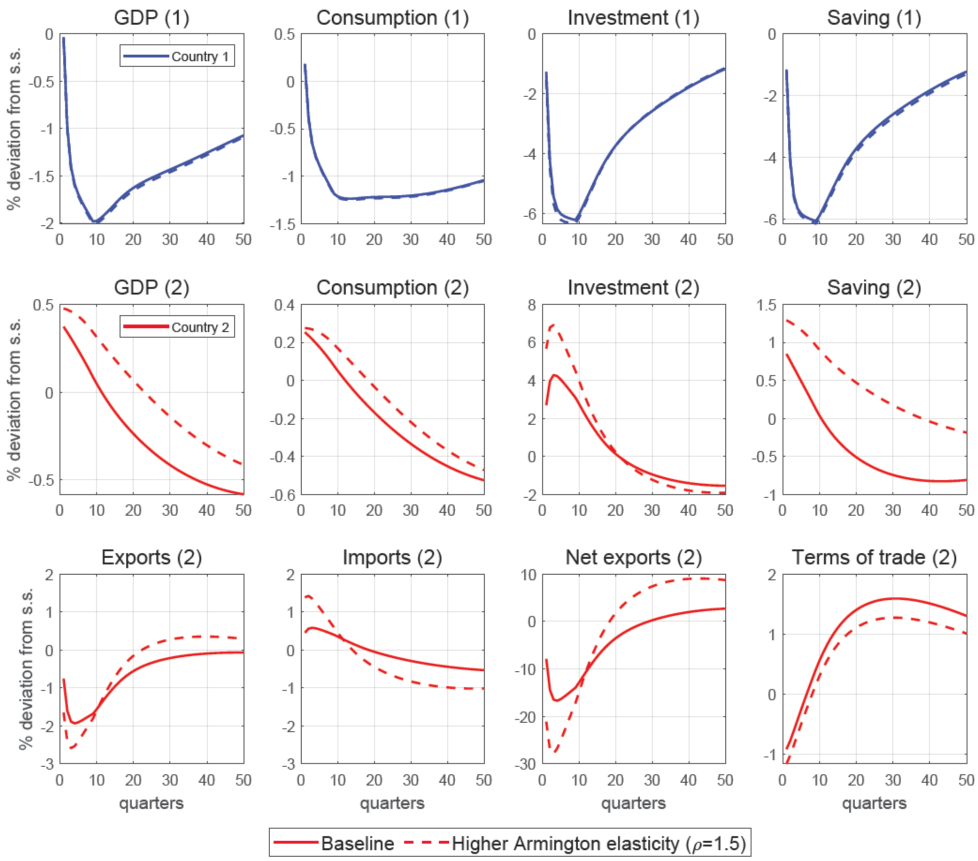
The Armington elasticity ρ is a critical parameter in quantitative analyses of open-economy models, as it determines the degree of substitutability between domestic and imported products. Our baseline calibration sets ρ equal to 0.9, implying that domestic and imported products are complements. Many other studies follow Backus, Kehoe and Kydland (1994) in assuming imports are substitutes for domestic goods and setting $\rho = 1.5$. We reconsider our model under that alternative choice here. For comparability sake, we recalibrate its remaining parameters to maintain the fit to our calibration targets in section 4. This requires, at most, third decimal place changes in all but the following 6 parameters associated with the two countries' trade shares and GDP.

TABLE A1. Armington elasticity: altered parameters

ρ	θ_{h1}	θ_{m1}	θ_{h2}	θ_{m2}	z_1^*	z_2^*
1.5	0.962	0.941	0.787	0.774	2.268	3.256

Figure A1 compares our baseline results (solid lines) with those from the alternative model with $\rho = 1.5$ (dashed lines). There, we see that allowing greater substitutability between domestic and imported products amplifies the responses of exports and imports, and thus net exports, following the country-1 credit shock. These changes have a negligible impact on the aggregate responses in country 1 given the small calibrated share of trade in its GDP. For country 2, however, the larger adjustment in its trade balance is mirrored by a more pronounced initial investment rise. The repercussions of this moderate the country-2 GDP decline over many quarters, delaying its trough by 13 quarters; however, the ultimate fall in GDP differs by under 0.1 percent, and the half-life of the recovery is still 75 quarters. Thus, the main conclusions from section 5.1 are unaltered when we eliminate our complementarity assumption.

FIGURE A1. Country-1 credit shock without complementarity



A.2 Population and trade-share differences

We emphasized our countries' great size disparity in discussing the investment-led transmission of country 1's credit shock to country 2 (figures 4 - 6), and the absence of transmission in the reverse direction (figure 7) in section 5.1. Recall that, in identifying the United States and Canada as the model's economies, our baseline calibration set country 2's relative population size at $\Psi_2 = 0.07$ to ensure its steady-state GDP is 8.8 percent that in country 1. We consider the importance of that assumption here, solving an alternative version of our model with $\Psi_2 = 1$.

Given the substantial change to country 2's household and firm population size, recalibrating the model to our targets in section 4 implies large parameter adjustments confounding comparisons with the baseline results. Here, we hold remaining parameters unchanged, allowing the two countries' steady-state GDP and hours worked to move away from the calibration targets. Despite equal population sizes, country 2's steady-state GDP is still 42 percent that in country 1 in the alternative model, a result of its greater openness to international trade reflected in the calibrated trade shares.

FIGURE A2. Country-1 credit shock without population differences

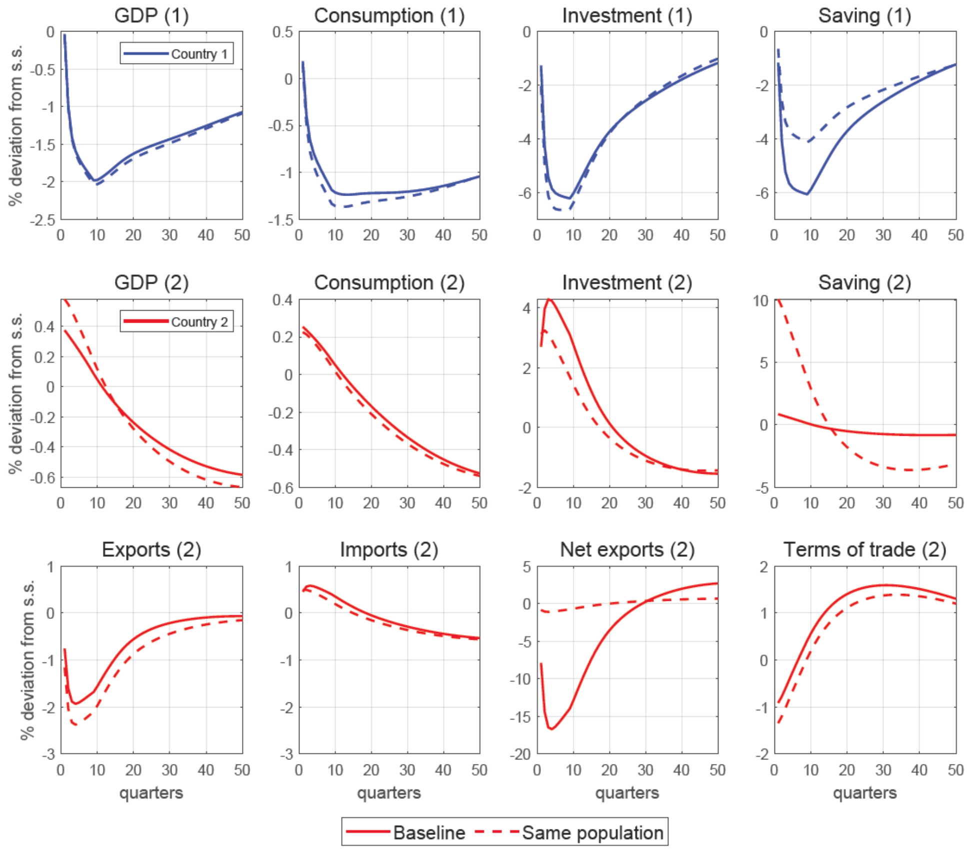
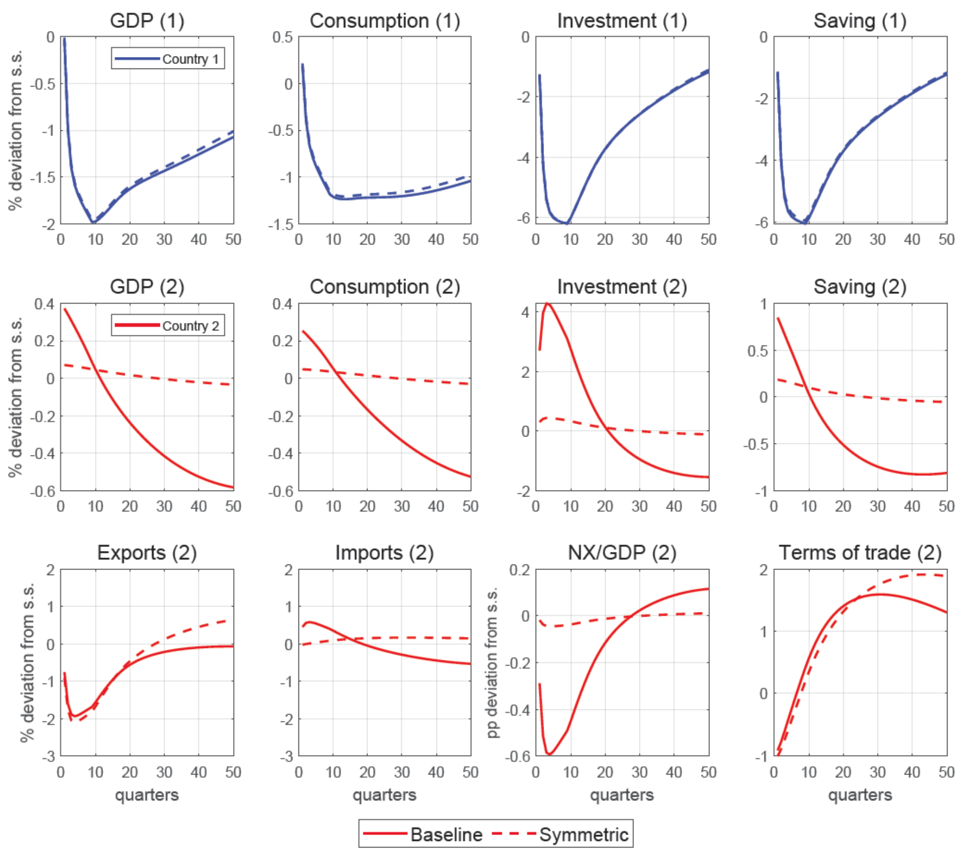


Figure A2 compares our baseline results to those from the model with equal populations. The greatest change in country-1 outcomes is a lesser fall in aggregate savings, and so a greater flow of residual savings to country 2. With its comparatively open trade partner now of its size, country-1 households reduce their savings by less even as domestic investment contracts by more.[1] The ramifications of this for country-2 investment are not so great as in the baseline model, however. These inflows are far smaller in relation to aggregates in the large country 2 economy; correspondingly, we see a smaller adjustment in its trade balance. Nonetheless, the rise in its relative return to investment is greater with more firms to spread the additional capital over. Measured productivity falls by less over the expansion, and the rise in investment now on its own offsets the fall in net exports. Effectively, its expanded size makes it easier for country 2 to take over a greater share of production while country-1's productivity is in decline; its GDP initially rises somewhat more, implying a greater contraction thereafter. While the path of country 1's recession and recovery is unaltered for reasons emphasized above, the gradualism in country 2's GDP responses lessens; its trough date arrives 1 year sooner, and the half-life of its recovery is 5 quarters shorter.

[1] Whereas country 1's trade balance is slightly negative (-0.003) in the baseline steady state, its greater preference for domestic goods than country 2's implies a small surplus, 0.089, in the model with equal populations.

On balance, figure A2 shows that eliminating population differences alters our results from section 5.1 very little, and the investment channel remains central to cross-country transmission of a financial shock.[2] In fact, country 2's small size is of far less consequence than is the comparatively large importance of trade in its GDP. To see what happens when that asymmetry is eliminated, we reset the remaining country-2 parameters to country-1 values. Figure A3 presents responses from the resulting model where the U.S. trades with a twin economy. Not surprisingly, when country 2 is as nearly a closed economy as is country 1, there is virtually no international transmission.

FIGURE A3. Country-1 credit shock with symmetric countries



A.3 Input-output linkages

Earlier studies have shown that input-output linkages in production amplify the transmission of various nominal and real shocks in closed- and open-economy environments.[3] Here, we explore the importance of these linkages in propagating a financial shock by considering a version of our model lacking them. More specifically, we eliminate the use of materials by resetting that productive

[2]The same comparison following a country-1 TFP shock shows consumption remains dominant in transmitting real shocks. These figures, and additional figures from the credit shock exercise, are available on request.

[3] See, for example, Alessandria and Choi (2014) for tariff changes, Luo and Villar (2023) for the price effects of demand and supply shocks, and Su (2023) for uncertainty shocks.

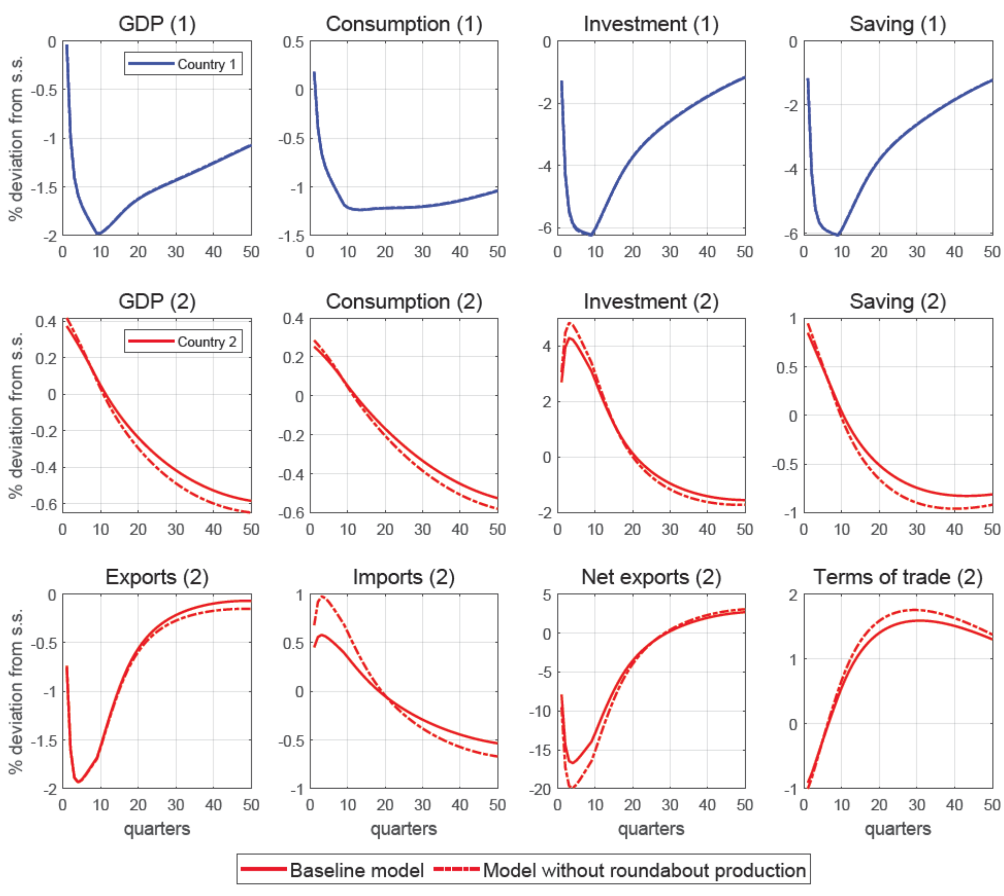
share to $\gamma = 0$. Such a large shift from the calibrated value, 0.43, implies changes in our economies too great to avoid a complete recalibration; Table A2 reports resulting parameter values differing beyond the third decimal place from their baseline counterparts in table 1.

TABLE A2. Input-output linkages: altered parameters

γ	α	ν	κ	θ_{h1}	θ_{h2}	z_1^*	z_2^*	σ_ε	k_{02}	b_{02}
0	0.287	0.60	2.396	0.986	0.814	1.122	1.815	0.052	1.479	0.512

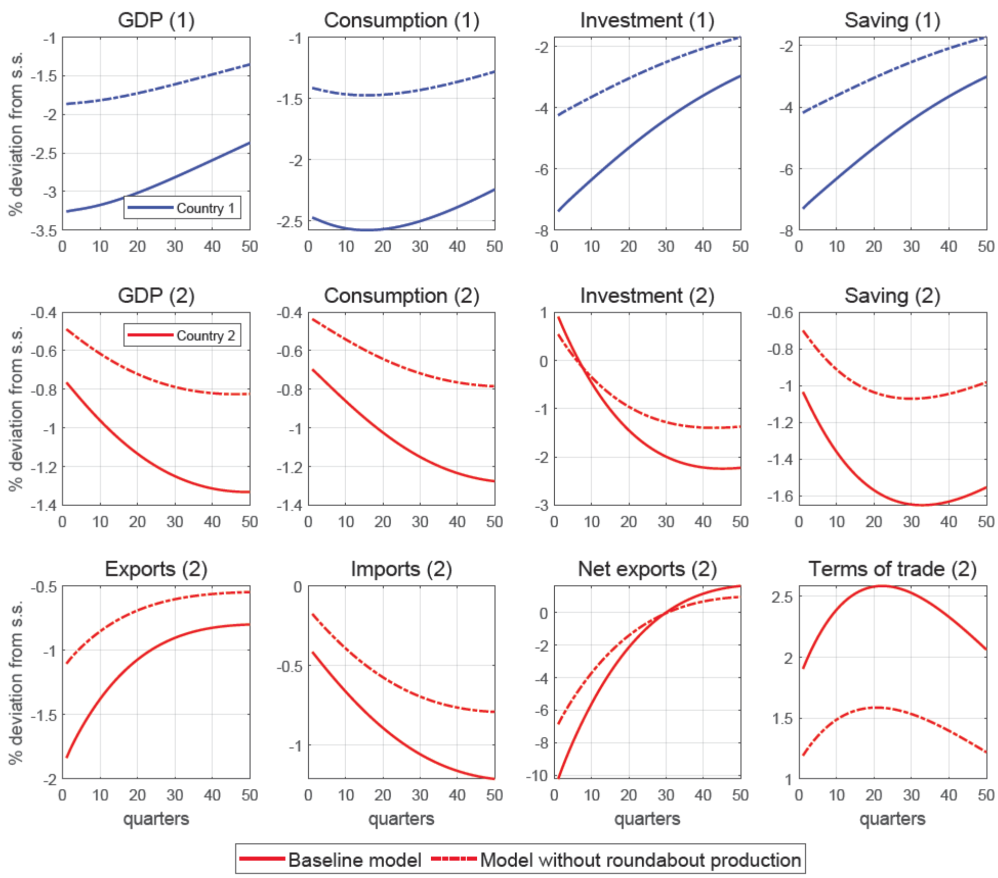
Figure A4 compares our baseline model's (solid line) responses following the country-1 financial shock to those in the alternative model with $\gamma = 0$. In light of prior findings in the literature, the top row of responses may come as a surprise. They indicate that roundabout production makes no difference for the domestic transmission of a credit shock to our large open economy. A more complete set of figures (available on request) shows country 1's measured TFP falls about twice as much in the alternate model, a natural consequence of our recalibration having raised the capital share parameter from 0.164 to 0.287; however, the only tangible consequence for country 1 is a greater rise in exports in those dates where the TFP gap is wide.

FIGURE A4. Country-1 credit shock without input-output linkages



Turning to international transmission, the figure's remaining rows show that the implied boost to country-2's imports implies a larger adjustment in its trade balance and correspondingly larger rises in its investment when input-output linkages are eliminated. Given the importance of those early-date rises for the correction that follows, its GDP subsequently falls by more and at a somewhat faster pace. In our model, imported final goods are used directly for capital investment, whereas imported intermediate inputs are further processed through domestic production. When materials are made redundant, only final goods are traded, thus facilitating the investment reallocation from country 1 to country 2 while country 1's credit-induced misallocation is worsening.

FIGURE A5. Country-1 TFP shock without input-output linkages



Finally, we confirm that the dynamic effects of production linkages in our model are consistent with the findings of prior studies when the source of fluctuations is a real shock. Figure A5 compares responses to a one-percent country-1 productivity shock in the same two economies as above. In contrast to the results in figure A4, we see here that our baseline model's roundabout production structure significantly amplifies the transmission of real shocks, both domestically and abroad.